

**Hong Kong Baptist University**  
**Faculty of Science**  
**Department of Mathematics**

**Title (Units): ORBS 7270 Financial Calculus (2,2,0)**

**Course Aims:** This course introduces derivative pricing. Students will learn some well-known mathematical pricing models of financial instruments under no arbitrary principle. We first overview some fundamental probability, statistical knowledge and skills required for mathematical finance. Then, we shall introduce an important concept in financial pricing models, Martingale and Risk neutrality. In the incomplete market, Martingale probability measure provides an important method in obtaining a fair instrument price under no arbitrary principle. Then, we shall study three well-known approaches to model the dynamic of the financial instruments, they are Markov process, Poisson process and Brownian model. To highlight the practical relevance of the course materials we shall discuss a number of real-world case studies throughout the course.

**Prerequisite:** None

**Prepared by:** Michael Ng

**Course Intended Learning Outcomes (CILOs):**

Upon successful completion of this course, students should be able to:

No.	Course Intended Learning Outcomes (CILOs)
1	Recognize the basic concepts of mathematical probability on $\sigma$ -field
2	Identify the importance of Martingales in financial pricing models
3	Recognize the basic knowledge in Markov and Poisson process and Brownian motion
4	Develop a fundamental knowledge and skills required for mathematical finance
5	Develop analytical and critical thinking and applying that to solve real-world financial problems

**Teaching & Learning Activities (TLAs):**

CILO	TLAs will include the following:
1,2,3,4,5	Lectures with rigorous mathematical discussions and concrete examples. The lecturer will constantly ask questions in class to make sure that the majority of students are following the teaching materials.
1,2,3,4,5	Assignments to monitor both students' learning and mastering of the taught materials. In addition, common mistakes will also be addressed and analyzed.

**Assessment:**

No.	Assessment Methods	Weighting	CILO Addressed	Remarks
1	Continuous Assessment	30%	all	Assessments are designed to measure how well the students recognizing of the theory,

				techniques, and applications of financial calculus. The test is conducted to monitor the students' recognizing of the theory, techniques and skills taught in the class. This may involve, but not limited to, in class discussions of rigorous technical problems and their solutions.
2	Final Examination	70%	all	Final Examination questions are designed to see how far students have achieved their intended learning outcomes. Students should have a thorough recognizing on essential knowledge in financial modeling and apply them correctly in various real life problems.

### Course Intended Learning Outcomes and Weighting:

Content	CILO No.	Teaching (in hours)
1. Review of Probability	1,4,5	2
2. Conditional Expectation	1,4,5	2
3. Martingales	2,4,5	9
4. Markov Chains	3,4,5	6
5. Poisson Process and Brownian Motion	3,4,5	8

**Textbook:** Zdzislaw Brzezniak and Tomasz Zastawniak. (2000). Basic Stochastic Processes. (3rd ed.). Springer.

**References:** Gregory Lawler. (2000). Introduction to Stochastic Processes. Chapman and Hall/CRC.  
Rick Durrett. Essentials of Stochastic Processes.  
Sheldon Ross. Stochastic Processes. (2nd ed.). Wiley.

### Course Content in Outline:

<u>Topic</u>	<u>Hours</u>
I. Review of Probability Theory on $\sigma$ -field	2
II. Conditional Expectation on	2
1. Financial Event	
2. Discrete Random Variable - E.g. Credit Rating	
3. Arbitrary Random Variable - E.g. Exchange Rate, Interest Rate	
4. $\sigma$ -field - Collection of Events of Interest	
III. Martingales for Derivatives Pricing	9
1. Martingales in Discrete Time -	
2. Filtrations - Financial Knowledge Sets	
3. Martingale Convergence Theorem - Finite Instrument Values	

4.	Optimal Stopping Time - Optimal Investment Quitting Time	
IV.	Markov Chains for Derivatives Modeling	6
1.	Definition: Markov Property for Modeling Derivative Movements	
2.	Classification of State - Financial Outcome Categories	
3.	Long-Time Behavior of Markov Chains - Stationary Property of Outcomes	
V.	Poisson Process and Brownian Motion for Modeling Financial Uncertainty	8
1.	Poisson Process - Modeling the Occurrence Frequency of Independent Events	
2.	Poisson Process Starts from Scratch at Time $t$	
3.	Brownian Motion – Modeling Independent Financial Increment	
4.	Sample Paths: Wiener Process for Financial Derivatives	