Determination Of Unknown Source Function In A Parabolic Equation

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In this paper we solve the following inverse problem of finding unknown source function in a parabolic equation: :

Find the temperature $u(x,t)$ and the heat source $f(x)$ which satisfy the equation

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}(k(x)\frac{\partial u}{\partial x}) + f(x), \qquad 0 < x < l, \qquad 0 < t \leqslant T \qquad (1)$$

subject to boundary conditions

$$u(0,t) = h_0(t) \qquad u(l,t) = h_l(t), \qquad 0 \leqslant t \leqslant T \qquad (2)$$

initial condition

$$u(x,0) = u_0(x), \qquad 0 \leqslant x \leqslant l \qquad (3)$$

and the overspecified condition

$$u(x,T) = \varphi(x), \qquad 0 \leqslant x \leqslant l \qquad (4)$$

The inverse problem (1)-(4) of determining an unknown heat source $f(x)$ in the heat conduction equation has been considered in many theoretical papers,notably [1-3], where the existence of smooth solution of the inverse problem (1)-(4) was studied. The unique solvability of this problem with $k(x) \equiv 1$ was established in [4]. One approach to solve this problem, which is referred to in the literature as the method of output least squares, is to assume that the unknown heat-source function $f(x)$ is of a specific functional form depending on some parameters and then seek to determine optimal parameter values which minimize an error functional based on the overspecified data. However, this approach has the drawback that it is usually not evident that the solution of the optimization problem solves the original inverse problem. In this work, by introducuing a new variable, we reformulate the problem as a nonclassical parabolic equation with the involvement of a trace -type! functional as a source term. After that we apply the Fixed Point Projection method to solve the reformulated problem. Some numerical examples are presented.

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References

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