The TTF Method For The Inverse Problem Of Finding Unknown Source Function In A Heat Equation

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In this work we consider the problem of structural identification of an unknown source term in a heat equation subject to the specification of the solution at the boundary. This problem is described by the following inverse problem: Find $u = u(x,t)$ and $F = F(u)$ which satisfy

$$u_t(x,t) = u_{xx}(x,t) + F(u(x,t)), \quad (x,t) \in Q_T = (0,1) \times (0,T), \tag{1}$$

$$u(x,0) = 0, \quad x \in (0,1), \tag{2}$$

$$u_x(0,t) = g(t), \quad t \in (0,T), \tag{3}$$

$$u_x(1,t) = 0, \quad t \in (0,T), \tag{4}$$

subject to the overspecification

$$u(0,t) = f(t), \quad t \in (0,T) \tag{5}$$

where $f(t)$ and $g(t)$ are known functions.

In the context of heat conduction and diffusion when u represents temperature and concentration the unknown function $F(u)$ is interpreted as a heat and material source, respectively, while in a chemical or biochemical application F may be interpreted as a reaction term.

The approach of the TTF method to the problem is to use the overspecified condition (5) to eliminate the unknown function from the equation (1). The resulting problem then has the form of a standart boundary value or initial boundary value problem but having coefficients which are functionals of the unknown solution. Thus the overspesified data is measured on the boundary then these functionals depend on restriction to the boundary of the solution (i.e. they will be "trace-type functionals"). After solving this TTF (Trace-type functional) problem, the unknown function can be determined using the overspecified conditions. The usual numerical methods such as finite diffrence and finite element can be used for numerical solution of TTF problem.

In this work we apply TTF method for the solution of considered problem and show effectiveness of this method by comparison with other methods.