On A Property Of Laplace Transforms Of Some Probability Densities

Katsuo Takano
DEPARTMENT OF MATHEMATICS, IBARAKI UNIVERSITY, JAPAN
ktaka@mx.ibaraki.ac.jp

It is known that all of probability distributions with density of normed product of the Cauchy densities such as
\[ f(a, b; x) = \frac{c}{(a^2 + x^2)(b^2 + x^2)}, \quad (0 < a < b) \]
are infinitely divisible. But it seems that it is not known if a probability distribution with density of normed product of the multi-dimensional Cauchy densities is infinitely divisible or not. In this talk we show a conjecture on the infinite divisibility of some probability distributions with density of normed product of odd dimensional Cauchy densities, namely,
\[ f(a, b; x) = \frac{c}{(a^2 + |x|^2(d+1)/2)(b^2 + |x|^2(d+1)/2)}, \quad (1) \]
where \( c \) is a normalised constant and
\[ 0 < a < b; \quad x = (x_1, x_2, \cdots, x_d) \in \mathbb{R}^d. \]

In this talk we assume the dimension \( d \) is an odd integer. We should note that the density \( f(a, b; x) \) can not be decomposed to a sum of partial fractions in the same way as in the 1-dimensional case and the speaker shows that we can overcome this difficulty. Making use of the formula
\[ K_\nu(z) = \frac{1}{2} \left( \frac{1}{2} \right)^\nu \int_0^\infty \exp\left\{-t - \frac{z^2}{4t}\right\} \frac{dt}{t^{\nu+1}}, \quad (2) \]
we obtain a Laplace-Stieltjes transform for the general odd dimensional case,
\[ \zeta(d, s) = \frac{c \pi^{d/2}}{\{(b^2 - a^2)^{d+1}\} \sum_{l=0}^\infty \frac{(-1)^{2l-j}(l+j)!}{(b^2 - a^2)^j} \left( \frac{l}{j} \right) 2^{j+1/2}}. \]
\[ \{a^{2j-1} K_{(j-1)+1/2}(2a\sqrt{s}) + (-1)^{j+1+j} b^{2j-1} K_{(j+1)+1/2}(2b\sqrt{s}) \}. \quad (3) \]