Parameter Estimation for Exponential Sums by Approximate Prony Method

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An important problem of digital signal processing is the so-called frequency analysis problem: Determine the different frequencies $\omega_j \in [0, \pi]$, the coefficients α_j, $\beta_j \in \mathbb{R}$, and the parameter $M \in \mathbb{N}$ from the sampled data $f(k)$ $(k = 0, \ldots, 2N)$ of a signal

$$f(x) = \frac{\alpha_0}{2} + \sum_{j=1}^{M} \big(\alpha_j \cos(\omega_j x) + \beta_j \sin(\omega_j x)\big) \quad (x \in \mathbb{R}).$$

This is a nonlinear inverse problem which can be simplified by original ideas of G. de Prony (1795).

In the talk, we report on new results of an approximate Prony method. The classical Prony method is notorious for its sensitivity to noise such that numerous modifications were attempted to improve its numerical behavior. Our results are based on papers of G. Beylkin and L. Monzón (Appl. Comput. Harmon. Anal. **19** (2005), 17-48). The nonlinear problem of finding the frequencies and coefficients can be split into two problems. To obtain the frequencies, we solve a singular value problem of the rectangular Hankel matrix $\mathbf{H} = \big(f(k+l)\big)_{k,l=0}^{2N-L,L}$ and find the the frequencies via roots of a convenient polynomial of degree L. To obtain the coefficients, we use the frequencies to solve a linear Vandermonde–type system. In contrast to G. Beylkin and L. Monzón, we apply matrix perturbation theory such that we can describe the numerical behavior of the approximate Prony method in detail. Numerical experiments show the performance of our method.