A S S I G N M E N T 2

Due date, Thursday, 15 March 2018

1. Consider the experiment of tossing two fair dice. Let A denote the event of an odd total, B the event of a 'one' on the first die and C the event of a total of seven. Are A and B, A and C, and B and C independent? Are A, B and C independent?

2. Suppose that you are at a party, in a hall filled with people. What is the minimum number of people do you think have to be present before the probability that at least two people have the same birthday is not less than 1/2? Having the same birthday here means the month and the day must match; the year is irrelevant. We also assume that no one in the hall has the birthday on February 29.

   Hint: The answer will be one of the following: 21, 22, 23 and 24.

3. A hand of five cards is to be dealt at random without replacement from an ordinary deck of 52 playing cards. What is the conditional probability of an all-spade hand, given that there are at least four spades in the hand?

4. Suppose each of two balls in an urn can be either red, black, or green with probability 1/3. A ball is chosen at random from the urn, and it is green. Now the green ball is replaced to the urn . What is the probability that the next ball chosen at random from the same urn is green?

5. There are three cards. Each card has a mark on each side. One card has a red mark on each side, one has a black mark on each side and one has a red mark on one side and a black on the other. One of the cards and one of its sides is chosen at random so that mark on only that one side is visible. This mark is seen to be red. What is the probability that the other side is also red?

6. Three plants, C_1, C_2 and C_3, produce respectively 10%, 50% and 40% of a company's output. Although plant C_1 is a small plant, it is believed that only 1% of its products are defective. The other two, C_2 and C_3, are worse and produce items that are 3% and 4% defective, respectively. All products are sent to a central warehouse. One item is selected at random and observed to be defective. What is the conditional probability that it comes from C_1?

7. (Optional bonus question: There will be no such kind of questions in the examination.) Given that events A and B are independent and that $\Pr(A \setminus B) = p_1$ and $\Pr(B \setminus A) = p_2$, where $0 \le p_1, p_2 \le p_1 + p_2 \le 1$ and $A \setminus B$ means the outcomes in A but not in B. Find $\Pr(A \cap B)$.

8. (Optional bonus question: There will be no such kind of questions in the examination.) Prove *mathematically* that if one of these four pairs of events, A and B, A^c and B, A and B^c, or A^c and B^c, is independent, then all fours pairs are independent. (The set A^c is the complement of the set A.)