A S S I G N M E N T   3

Due date: 12 April 2018

1. Suppose on average only five vehicles pass a small but the only one window of a café every hour. Imagine that a statistics professor is sitting in the café, enjoying a cup of cappuccino and looking at the window. What is the probability that he observes at least three vehicles passing the window within 10 minutes when he is in the café?

2. In a population of 60 light bulbs, it is known that there are 5 defective bulbs. A customer who does not know the proportion of defective bulbs chooses 10 bulbs at random for testing. If at most 1 of the sampled bulbs are defective, he accepts the lot. Find the probability of acceptance.

3. If $X \sim \mathrm{N}(3, 9)$, find (a) $\Pr(2 < X < 5)$, (b) $\Pr(X > 0)$, (c) $\Pr(|X - 3|) > 6)$, (d) α where $\Pr(X > \alpha) = 0.01$, and (e) β where $\Pr(X < \beta) = 0.8$.

4. Suppose that earthquakes occur in Japan at a rate 2 times per week.
   (a) What is the probability that at least 3 earthquakes occur during the next 2 weeks?
   (b) What is the probability that the time starting from now until the next earthquake is longer than 4 weeks?

5. A person gets \$40 if he or she gets three heads in three random and independent flips of a balanced coin; otherwise, there is no reward. How much should the person pay to play this game so as to make it fair?

6. The ideal size of lectures in a statistics course is 30. The AR of a university, knowing from past experience that on the average only 2/3 of the students will actually attend, allows 45 students to enrol. Assuming that whether a student attends the lectures or not does not depend on other students and the probability of which is the same for each student, use the normal distribution to find an approximate probability that more than 30 students attend the lectures.

7. An expert witness in a paternity suit testifies that the length (in days) of pregnancy (i.e. the continuous time from impregnation to the delivery of the child) is approximately normally distributed with parameters $\mu = 270$ and $\sigma^2 = 100$. The defendant in the suit is able to prove that he was out of the country during a period that began 290 days before the birth of the child and ended 240 days before the birth. What is the probability that the mother could have had the very long or very short pregnancy indicated by the testimony?

8. Derive mathematically the mean and the variance of a Poisson distribution with parameter λ. Hint: You can use the fact, without proof, that

$$\sum_{\text{all possible } x} \Pr(X = x) = 1.$$

9. A lake contains a very large number of fish. The length of a fish caught may be taken as a random variable, normally distributed with mean 20 cm and a standard deviation 4 cm. A fisherman keeps any fish he catches which is more than 25 cm long, returning others to the lake.

   (a) To double the yield, the fisherman keeps any fish he catches which is more than x cm long instead of 25 cm. Find x.

   (b) If the standard deviation of the length of a fish caught is y cm instead of 4 cm, the fisherman can still double his yield by keeping only those fish longer than 25 cm long. Find y.

   (c) If a group of 10 fishermen, all keeping only fish more than 25 cm long, each catch 10 fish, find the probability that the mean number of fish kept is 1 or more per fisherman.

10. (a) Let the observed value of the mean of a random sample of size 20 from $\mathrm{N}(\mu, 80)$ be 81.2. Find a 95% confidence interval for μ.

    (b) If the value 80 in (a) is not the true variance but only an estimate by the sample variance, find a 95% confidence interval for μ.

11. Find n such that we can construct a 90% confidence interval of length at most 2 from a random sample of size n from $\mathrm{N}(\mu, 9)$.