STAT 3710 Project 1

1. In norther climates, roads must be cleared of snow quickly following a storm. One measure of storm severity is x_1 = its duration in hours, while the effectiveness of snow removal can be quantified by x_2 =the number of hours crews, men, and machine, spend to clear snow. Here are the results for 25 incidents in Wisconsin.

   (a) Find the sample mean and variance of the difference $x_2 - x_1$ by the first obtaining the summary statistics.

   (b) Obtain the mean and variance by first obtaining the individual values $x_{j2} - x_{j1}$ for $j = 1, 2, \ldots, 25$ and then calculating the mean and variance. Compare these values with those obtained in part a.

   (c) Comment on any possible outliers in in scatter plot of the original variables.

   (d) Determine the power transformation $\hat{\lambda}_1$ the makes the x_1 values approximately normal. Construct a Q-Q plot of the transformed observations.

   (e) Determine the power transformation $\hat{\lambda}_2$ the makes the x_2 values approximately normal. Construct a Q-Q plot of the transformed observations.

   (f) Determine the power transformation for approximate bivariate normality.

2. Measurements of x_1 = stiffness and x_2 = bending strength for a sample of $n = 30$ pieces of a particular grade of lumber are given in Table 5.11. The units are pounds/(inches). Using the data in the table

   (a) Construct and sketch a 95% confidence ellipse for pair $[\mu_1, \mu_2]'$, where $\mu_1 = \mathrm{E}X_1$ and $\mu_2 = \mathrm{E}X_2$.

   (b) Suppose $\mu_{10} = 2000$ and $\mu_{20} = 10,000$ represent "typical" values for stiffness and bending strength, respectively. Given the result in (a), are the data in Table 5.11 consistent with these values ? Explain.

   (c) Is the bivariate normal distribution a viable population model? Explain with reference to Q-Q plots and a scatter diagram.

Table 3.2 Snow Data

x_1	x_2	x_1	x_2	x_1	x_2
12.5	13.7	9.0	24.4	3.5	26.1
14.5	16.5	6.5	18.2	8.0	14.5
8.0	17.4	10.5	22.0	17.5	42.3
9.0	11.0	10.0	32.5	10.5	17.5
19.5	23.6	4.5	18.7	12.0	21.8
8.0	13.2	7.0	15.8	6.0	10.4
9.0	32.1	8.5	15.6	13.0	25.6
7.0	12.3	6.5	12.0		
7.0	11.8	8.0	12.8		

3. The tail lengths in millimeters (x_1) and wing lengths in millimeters (x_2) for 45 male hook-billed kites are given Table 6.11. Similar measurements for female hook-billed kites are given in Table 5.12

   (a) Plot the male hook-billed kite data as a scatter diagram and (visually) check for outliers.

   (b) Test for equality of mean vectors for the populations of male and female hook-billed kites. Set $\alpha = 0.05$. If $\boldsymbol{\mu}_1 - \boldsymbol{\mu}_2 = 0$ is rejected, find the linear combination most responsible for the rejection of H_0. (You may want to eliminate any outliers found in Part a for the male hook-billed kite data before conducting this test. Alternatively, you may want to interpret $x_1 = 284$ for observation 31 as a misprint and conduct the test with $x_1 = 184$ for this observation. Does it make any difference in this case how observation 31 for the male hook-billed kite data is treated?)

   (c) Determine 95% confidence region for $\boldsymbol{\mu}_1 - \boldsymbol{\mu}_2$ and 95% simultaneous confidence interval for the components of $\boldsymbol{\mu}_1 - \boldsymbol{\mu}_2$.

   (d) Are male or female birds generally larger ?

Table 5.11 Lumber Data

x_1 (Stiffness: modulus of elasticity)	x_2 (Bending strength)	x_1 (Stiffness: modulus of elasticity)	x_2 (Bending strength)
1232	4175	1712	7749
1115	6652	1932	6818
2205	7612	1820	9307
1897	10,914	1900	6457
1932	10,850	2426	10,102
1612	7627	1558	7414
1598	6954	1470	7556
1804	8365	1858	7833
1752	9469	1587	8309
2067	6410	2208	9559
2365	10,327	1487	6255
1646	7320	2206	10,723
1579	8196	2332	5430
1880	9709	2540	12,090
1773	10,370	2322	10,072

Source: Data courtesy of U.S. Forest Products Laboratory.

Table 5.12 Bird Data

x_1 (Tail length)	x_2 (Wing length)	x_1 (Tail length)	x_2 (Wing length)	x_1 (Tail length)	x_2 (Wing length)
191	284	186	266	173	271
197	285	197	285	194	280
208	288	201	295	198	300
180	273	190	282	180	272
180	275	209	305	190	292
188	280	187	285	191	286
210	283	207	297	196	285
196	288	178	268	207	286
191	271	202	271	209	303
179	257	205	285	179	261
208	289	190	280	186	262
202	285	189	277	174	245
200	272	211	310	181	250
192	282	216	305	189	262
199	280	189	274	188	258

Source: Data courtesy of S. Temple.

Table 6.11 Male Hook-Billed Kite Data

x_1 (Tail length)	x_2 (Wing length)	x_1 (Tail length)	x_2 (Wing length)	x_1 (Tail length)	x_2 (Wing length)
180	278	185	282	284	277
186	277	195	285	176	281
206	308	183	276	185	287
184	290	202	308	191	295
177	273	177	254	177	267
177	284	177	268	197	310
176	267	170	260	199	299
200	281	186	274	190	273
191	287	177	272	180	278
193	271	178	266	189	280
212	302	192	281	194	290
181	254	204	276	186	287
195	297	191	290	191	286
187	281	178	265	187	288
190	284	177	275	186	275

Source: Data courtesy of S. Temple.