Recent Progress in Finite Element Heterogeneous Multiscale Methods

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In this talk we discuss hierarchical finite element methods (FEMs) for multiscale elliptic problems constructed in the framework of the heterogeneous multiscale method (HMM) introduced by W. E and B. Engquist. For multiscale partial differential equations (PDEs), the HMM discretizes the physical problem directly by a macroscopic model. The unknown data of the macromodel are extracted on the fly by testing the microstructure on sampling domains.

Many fundamental numerical and modeling issues arise in such a hierarchical coupling of solvers as the propagation of errors of a micro model to the macroscale [1] or the complexity of the overall numerical methods [2]. One strength of the HMM is its versatility, allowing much freedom to couple different types of solvers at different scales. This will be illustrated by discussing a discontinuous Galerkin HMM for multiscale elliptic problems [3]. Another strength of the framework of the HMM is the possibility to use the structure of traditional finite element solvers to design computer codes for multiscale problems. This will be briefly illustrated by discussing a recently proposed finite element implementation for homogenization problems [4].

References

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