Discontinuous Galerkin Methods and Local Time Stepping for Second-Order Wave Equations

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The accurate and reliable simulation of wave phenomena is of fundamental importance in a wide range of engineering applications such as fiber optics, wireless communication, radar and sonar technology, and non-invasive testing. To address the wide range of difficulties involved, we consider symmetric interior penalty discontinuous Galerkin (IP-DG) methods, which easily handle elements of various types and shapes, irregular non-matching grids, and even locally varying polynomial order. Moreover, in contrast to standard (conforming) finite element methods, IP-DG methods yield an essentially diagonal mass matrix; hence, when coupled with explicit time integration, the overall numerical scheme remains truly explicit in time.

To circumvent the severe stability (CFL) condition imposed on the time step by the smallest elements in the mesh, we propose local time-stepping schemes, which allow arbitrarily small time steps where small elements in the mesh are located. When combined with the symmetric IP-DG discretization, the resulting fully discrete scheme is explicit and exactly conserves a discrete energy. Starting from the standard second order “leap-frog” scheme, time integrators of arbitrary order of convergence are derived. Numerical experiments illustrate the usefulness of these methods and validate the theory.