Continuous and Discrete Helmholtz-type Decompositions

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The term Helmholtz-type decomposition of $\mathbf{H}(\mathbf{curl}, \Omega)$ refers to stable splittings of the form

$$\mathbf{H}(\mathbf{curl}, \Omega) = (H^1(\Omega))^3 + \mathbf{grad} H^1(\Omega).$$

First mentioned in a work by Birman and Solomyak [1], splittings of this type have quickly become a key tool in both the theoretical and numerical analysis of spaces of **curl**-conforming vectorfields and related variational boundary value problems. They and their discrete counterparts proved instrumental in

- the investigation of extension theorems and trace spaces for $\mathbf{H}(\mathbf{curl}, \Omega)$ [2],
- the derivation and regularity and compactness results [4],
- the analysis of boundary integral formulations related to Maxwell's equations [3,8],
- the development of weighted regularization techniques for Lagrangian finite element schemes for electromagnetics boundary value problems [5],
- the design of auxiliary space preconditioners for $\mathbf{H}(\mathbf{curl}, \Omega)$-elliptic variational boundary value problems [9].
- the subspace correction theory of multigrid methods for edge elements [6,7,10].

First, my presentation will outline proofs of the existence of continuous Helmholtz-type decompositions. Then some of the applications listed above will be addressed. Next, I am going to introduct discrete Helmholtz-type decompositions. Finally, their use for finite element theory will be elaborated.

References

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