On the Geometric Aspect Homogenization

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It is known that solutions of elliptic (parabolic) divergence form equations with L^∞ (bounded) gain one order of regularity in harmonic (caloric) coordinates and that numerical homogenization algorithms can be deduced for situations where the medium is characterized by a continuum of scales. In this talk we will look into the geometric aspect of homogenization theory. For elliptic equations we show that a scalar conductivity is in one to one correspondence with a convex function and that the numerical homogenization of the former is equivalent to the linear interpolation of the latter. We show how this observation can be used to translate inverse homogenization from the search of a (optimal/penalized) solution within a nonlinear space into the search of a (optimal/penalized) solution within the linear space.

For vectorial (elasticity) equations we show that numerical homogenization with a continuum of scales is possible through the introduction/application of a new analytical/geometric inequality allowing for the approximation of regular (H^1) gradients fields by linear combinations of irregular (L^2) divergence free fields. Parts of this talk are joint works with Lei Zhang, Roger Donaldson, Leonid Berlyand, Mathieu Desbrun and Yiying Tong.