

Adaptive Tensor Product Wavelet Methods

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Sparse grid or hyperbolic cross approximations based on a tensor product of multiscale bases converge with a rate equal to the worst rate of the individual factors. In particular, the rate does not deteriorate with the number of factors. This result holds true when the function to be approximated has bounded mixed derivatives of certain higher order. In cases of interest, this regularity condition is not necessarily fulfilled as we will demonstrate for the example of Poisson's problem with a general smooth right hand side.

Above regularity condition can be drastically reduced by considering best N -term approximation from the tensor product basis. Moreover, when the function to be approximated is given as the solution of an operator equation, the rate of best N -term approximation can be realized by adaptive wavelet methods. As applications, we will discuss in some detail the solution of Poisson's problem in high space dimensions, and that of parabolic evolution equations in simultaneous space-time variational formulation.