Numerical Analysis of Fokker–Planck Equations with Unbounded Drift

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The kinetic theory of dilute polymers is a fertile source of high-dimensional, possibly degenerate, Fokker–Planck type drift-diffusion equations with unbounded drift terms. A relevant feature of the class of equations under consideration from the viewpoint of mathematical analysis and numerical approximation is the presence of an unbounded drift coefficient, involving a smooth convex potential U that is equal to $+\infty$ along the boundary ∂D of the computational domain D. Using a symmetrization of the differential operator based on the Maxwellian M corresponding to U, which vanishes along ∂D, we remove the unbounded drift coefficient at the expense of introducing a degeneracy, through M, in the principal part of the operator. We show the existence of weak solutions to the initial-boundary-value problem in suitable Maxwellian-weighted Sobolev spaces, and develop a fully discrete spectral Galerkin method for such degenerate Fokker–Planck equations that exhibits optimal-order convergence in the Maxwellian-weighted H^1 norm on D. We will also discuss the convergence analysis of an operator-splitting algorithm for high-dimensional Fokker–Planck equations, and the convergence of a general class of Galerkin methods for coupled Navier–Stokes–Fokker–Planck systems.