Compact High Order Finite Volume Methods With and Without Intrinsic Constraints

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We consider high order finite volume methods for the numerical solution of conservation laws with and without intrinsic constraints. Classical examples are the Euler equations of gas dynamics or the equations of magnetohydrodynamics (MHD) where the divergence condition for the magnetic field comes as intrinisic constraint.

The first part of the talk introduces a new class of limiter functions based on double-logarithmic reconstruction. The result is a class of efficient third-order schemes with a compact three point stencil. The new methods handle discontinuities as well as local extrema within the standard semi-discrete TVD-MUSCL framework and an explicit TVD Runge-Kutta time marching scheme. In addition, computational efficiency is enhanced due to large allowable Courant numbers ($CFL < 1.6$), as shown by von-Neumann stability analysis.

The second part of the talk uses the theory of flux distributions to incorporate an intrinsic differential constraint of the equations into any high order finite volume method. This leads to a seamless and local contraint preservation. The higher order extension is performed for the divergence constraint in MHD on the basis of a fourth order divergence operator that implies a rich set of flux distributions. In the final scheme this fourth order accurate discrete divergence operator is preserved to round-off errors when applied to the cell averages of the magnetic field.

Combining both parts we will present numerical experiments for Euler equations and MHD that demonstrate the properties of the new scheme like improvement of shock resolution, high accuracy for smooth functions, local divergence preservation and computational efficiency.