On Numerics for Stochastic PDEs

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Partial differential equations with stochastic inputs are a suitable tool to describe systems whose parameters are not completely determined, either because of measurement errors or intrinsic lack of knowledge on the system. Here the goal is to approximate statistical moments of functionals depending on the solution.

We first discuss some recent results on the construction of quasi optimal stochastic Galerkin and stochastic Collocation approximations for linear elliptic equations with random coefficients. To this end, we develop a strategy to construct optimal sparse grids, based on an a priori selection of the most profitable hierarchical surpluses to be included in the sparse grid. The selection relies on sharp estimates of the error and work contributions (profit) of each hierarchical surplus to be added. In particular, the error contribution includes a term that accounts for the decay in each of the random variables separately, determined numerically through inexpensive 1D analyses, and a second term that accounts for the coupling among the random variables.

Finally, we propose adaptive Multilevel Monte Carlo methods based on non uniform refinements to accelerate the convergence of a standard, single level, Monte Carlo method.