Using Gabor dictionaries in a $TV - l^{\infty}$ model, for denoising

Tieyong Zeng, Françoise Malgouyres
LAGA/L2TI, Université Paris 13
99 avenue Jean-Batiste Clément, 93430 Villetaneuse, France
E-mail: {zeng.malgouyres}@math.univ-paris13.fr

Abstract
The goal of this poster is to report on experiments where we use Gabor dictionaries in a $TV - l^{\infty}$ model for denoising. This allows many possible choices. Our conclusions are that the choice of the dictionary mostly impact the restoration of textures. Moreover, for most images, better results are obtained when the Gaussian term of the Gabor filters is close to isotropic.

Inverse Problem
Problem : \[ u_0 = u + b \]
\(b \in \mathbb{R}^{N^2}\), Gaussian white noise, standard variation \(\sigma\)
\(u_0 \in \mathbb{R}^{N^2}\) image observed
\(u \in \mathbb{R}^{N^2}\) image want to recover.

Total Variation
The Total Variation is discretized as :
\[ TV(u) = \sum_{i,j=0}^{N-1} |\nabla(u)_{i,j}| \]
with
\[ \nabla(u)_{i,j} = (u_{i+1,j} - u_{i,j}, u_{i,j+1} - u_{i,j}) \]

Models
The famous Rudin-Osher Model is an algorithm for minimizing
\[ TV(u) + \lambda \|u - u_0\|_2 \]
Our Models :
\[ \text{minimize } TV(u) \text{ under the constraint } \|u - u_0\|_{\mathbb{D},\infty} \leq \tau \]
where \[ \|\cdot\|_{\mathbb{D},\infty} \text{ is defined by} \]
\[ \|w|_{\mathbb{D},\infty} = \sup_{\psi \in \mathbb{D}} |\langle w, \psi \rangle| \]
for a finite dictionary \(\mathbb{D} \subset \mathbb{R}^{N^2}\).

Key problem : how to design \(\mathbb{D}\)

- curvelet dictionary
- wavelet packet dictionary
- Our new approach Gabor dictionary

Gabor filter
\[ \hat{g}_{m,n}^\theta \phi = C_{\phi} \frac{\sin(2\pi f x)}{N} \]
where \(f\) and \(\theta \in \mathbb{R}\) and \(\phi \in \mathbb{R}^d\) need to be chosen, \(x = m \cos \theta + n \sin \theta, y = -m \sin \theta + n \cos \theta\) and \(C\) is such that the \(l^1\) norm of the features equal \(1/2\).

Example of Gabor filter

From features to dictionary
From the basic dictionary :
\[ \mathbb{F} = \{\psi_{m,n}^\theta\}_{0 \leq m,n < N} \]
Let : For any \(k \in \{1, \ldots, r\}\) and any indexes \((i,j) \in \{0, \ldots, N-1\}^2\), we denote
\[ \psi_{m,n}^{k,i,j} = \psi_{m,n-k,i,j} \]
where \([m,n] \in \{0, \ldots, N-1\}^2\).
Then we can create a translated-invariant dictionary
\[ \mathbb{D} = \{\psi_{k,i,j}\}, \text{for } 1 \leq k \leq r \text{ and } 0 \leq i,j < N\).\]

4 types of features
Sum of the Fourier transforms of the :
- First : Gabor I features
- Second : features with curvelet scaling
- Third : Gabor III features
- Forth : Gabor II features

Numerical Aspects
Using the idea of Penalty method : we minimize the unconstrained energy
\[ TV(u) + \lambda \sum_{\psi \in \mathbb{D}} \psi_i(u - u_0, \psi) \]
with
\[ \psi_i(t) = \langle w, \psi \rangle - \langle t, \psi \rangle \]
and for a large number \(\lambda\). In order to use Steepest descent algorithm, we need to compute the gradient of (4) :
\[ \nabla TV(u) + \lambda \sum_{\psi \in \mathbb{D}} \psi_i(u - u_0, \psi) \Psi \]
where \(\psi'_i\) denotes the derivative of \(\psi_i\).

Decomposition
To calculate :
\[ \langle (u, \psi_{m,n}), \psi_{m,n} \rangle_{1 \leq m,n < N} \text{ for } 1 \leq \gamma \leq \mathbb{F} \]
It’s :
\[ \langle (u, \psi_{m,n}), \psi_{m,n} \rangle_{1 \leq m,n < N} = \sum_{k=0}^{N-1} u_{m,n} \psi_{k,m,n-k} \]
It’s :
\[ u \ast \psi_{m,n} = \psi_{m,n-u} \]
One Fourier transform and \#\(\mathbb{F}\) inverse Fourier transforms.

Recomposition
Denoting \(\lambda = \{\lambda_{m,n}\}_{1 \leq m,n < N} \text{ and } 1 \leq \gamma \leq \mathbb{F} \)
and \(m = \psi_{m,n} \mathbb{F}^2\), the recomposition takes the following form
\[ T : \Lambda \in \mathbb{R}^{\#\mathbb{F}} = \sum_{\gamma \in \mathbb{F}} \lambda_{\gamma} \psi_{\gamma} \in \mathbb{R}^N \]
It’s
\[ T(\Lambda) = \sum_{\gamma \in \mathbb{F}} \lambda_{\gamma} \psi_{\gamma} \]
\#\(\mathbb{F}\) Fourier transforms and one inverse Fourier transform.

Experiments
We report on denoising experiments of the image "Barbara". The noise variance is \(\sigma = 20\). The twelve dictionaries have been tested. For each dictionary, we tuned the parameter \(\tau\) to obtain good visual results. We focus on three regions of the right image.

zone 1
PSNR for zone 1

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<th>medium</th>
<th>large</th>
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<tbody>
<tr>
<td>Gabor I</td>
<td>27.365</td>
<td>27.484</td>
<td>27.641</td>
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<tr>
<td>Gabor II</td>
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<td>27.169</td>
<td>26.859</td>
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<tr>
<td>curvelet</td>
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<td>27.171</td>
<td>27.019</td>
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<tr>
<td>Gabor III</td>
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zone 2
Left : Noisy zone 2 ; center : result for the medium "curvelet scaling" dictionary, PSNR = 21.7.

zone 3
PSNR

<table>
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<tr>
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<td>21.835</td>
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<tr>
<td>Gabor II</td>
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<tr>
<td>curvelet</td>
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<td>Gabor III</td>
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Gabor II V.S. ROF

More results are available at

Thank you!