Varying Coefficient Partial Differential Equations, with Applications to LIDAR Data

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Partial differential equation (PDE) models are commonly used to model complex dynamic systems in applied sciences such as biology and finance. The forms of these PDE models are usually proposed by experts based on their prior knowledge and understanding of the dynamic system. Parameters in PDE models often have interesting scientific interpretations, but their values are often unknown, and need to be estimated from the measurements of the dynamic system in the present of measurement errors. Most PDEs used in practice have no analytic solutions, and can only be solved with numerical methods. Currently, methods for estimating PDE parameters require repeatedly solving PDEs numerically with thousands of candidate parameter values, and thus the computational load is high. In this article, we propose methods to estimate parameters in PDE models, including those with varying coefficients: a parameter cascading method, a Bayesian approach and a kernel-based approach. In the first two methods, the underlying dynamic process modeled with the PDE model is represented via basis function expansion. In linear PDE, simulation studies show that the new methods are comparable, and both outperform other available methods in terms of estimation accuracy. The methods are demonstrated by estimating parameters in a PDE model from LIDAR data.