Chapter 1

1.1 Consider the amount function \( A(t) = t^2 + 2t + 3 \)
   a) Find the corresponding accumulation function \( a(t) \).
   b) Verify that \( a(t) \) satisfies the three properties of an accumulation function
   c) Find \( I_n \).

1.2 It is know that \( a(t) \) is of the form \( at^2 + b \). If $100 invested at time 0 accumulates to $172 at time 3, find the accumulated value at time 10 of $100 invested at time 5.

1.3 (a) Assume \( A(t) = 100 + 5t \). find \( i_5 \) and \( i_{10} \).
   (b) Assume that \( A(t) = 100(1.1)^t \), find \( i_5 \) and \( i_{10} \).

1.4 If \( A(4) = 1000 \) and \( i_n = .01n \) find \( A(7) \).

1.5 (a) At what rate of simple interest will $500 accumulate to $615 in 2\frac{1}{2} \) years ?
   (b) IN how many years will $500 accumulate to $630 at 7.8\% simple interest?

1.6 Simple interest of \( i = 4\% \) is being credited to fund. In which period is this equivalent to an effective rate of 2.5\%.

1.7 Assume that \( 0 < i < 1 \), show that: a) \((1 + i)^t < 1 + it \) if \( 0 < t < 1 \), b) \((1 + i)^t > 1 + it \) if \( t > 1 \).

1.8 It is known that $600 invested for two years will earn $264 in compound interest. Find the accumulated value of $2000 invested at the same rate of compound interest for three years.

1.9 The sum of the present value of 1 paid at the end of \( n \) period and 1 paid at the end of \( 2n \) is 1, Find \((1 + i)^{2n}\).
1.10 It is known that an investment of $500 will increase to $4000 at the end of 30 years. Find the sum of the present values of three payments of $10,000 each which will occur at the end of 20, 40, and 60 years.

1.11 The amount of interest earn on $A$ for one year is $336, while the equivalent amount of discount is $300. Find $A$.

1.12 Assuming that $0 < d < 1$, show that a) $(1 - d)^t < 1 - dt$ if $0 < t < 1$, b) $(1 - d)^t > 1 - dt$ if $t > 1$.

1.13 a) Express $d^{(4)}$ as a function of $i^{(3)}$. b) Express $i^{(6)}$ as a function of $d^{(2)}$.

1.14 Find the accumulated value of $\$100$ at the end of two years if the nominal annual rate of interest is 6% convertible quarterly.

1.15 Given that $i^{(m)} = .1844144$ and $d^{(m)} = .1802608$, find $m$.

1.16 Show that a) $\int_0^n \delta_t dt = -\log_e v^n$. b) $\int_0^n A(t) \delta_t dt = I_1 + I_2 + \cdots + I_n$.

1.17 Find the level effective rate of interest over a three year period which is equivalent to an effective rate of discount 8% the first year, 7% the second year, and 6% the third year.