Chapter 2

2.1 A family wishes to accumulate $50,000 in college education fund at the end of 20 years. If they deposit $1000 in the fund at the end of each the first 10 years and $1000 + X in the fund at the end of each the second 10 years, find X to be nearest dollar if the fund earns 7% effective.

2.2 The cash price of a new automobile is $10,000. The purchase is willing to finance the car at 18% convertible monthly and to make payments of $250 at the end of each month for four years. Find the down payment which will be necessary.

2.3 (a) Show that \( a_{m+n} = a_m + v^m a_n = v^n a_n + a_n \).
(b) Show that \( s_{m+n} = s_m + (1 + i)^m s_n = (1 + i)^n s_n + s_n \).
(c) Verbally interpret the results in (a) and (b).

2.4 You are given the following annuity values:
\[ a_7 = 5.153, a_{11} = 7.036, a_{18} = 9.180 \]
Find \( i \).

2.5 Find the present value of payment of $200 every six months starting immediately and continuing four years from the present, and $100 every six months thereafter through ten years from the present if \( i^{(2)} = .06 \).

2.6 A worker aged 40 wishes to accumulate a fund for retirement by deposit $1000 at the begin of each year for 25 years. Starting at age 65 the worker plans to make 15 annual withdraws at the beginning of each year. Assuming that all payments are certain to be made, find the amount of each withdraws starting at age 65 to the nearest dollar, if the effective rate of interest is 8% during the first 25 years, but only 7% thereafter.

2.7 Find \( \bar{a}_n \) if the effective rate of discount is 10%.

2.8 (a) Show that \( \bar{a}_n = a_n + 1 - v^n \).
(b) Show that \( \bar{s}_n = s_n - 1 + (1 + i)^n \).
(c) Verbally interpret the results in (a) and (b).
2.9 Payments of $100 per quarter are made from June 7, \( Z \) through December 7, \( Z + 11 \), inclusive. If the nominal rate of interest convertible quarterly is 6%:
   a) Find the present value on September 7, \( Z - 1 \).
   b) Find the current value on March 7, \( Z + 8 \).
   c) Find the accumulated value on June 7, \( Z + 12 \).

2.10 At an annual effective interest rate \( i \) it is known that:
   (i) The present value of 2 at the end of each year for \( 2n \) years, plus an additional 1 at the end of the first \( n \) years, is 36.
   (ii) The present value of an \( n \)-year deferred annuity-immediate paying 2 per year for \( n \) years is 6.
   Find \( i \).

2.11 Find the present value to the nearest dollar on January 1 of an annuity which pays $2000 every six months for five years. The first payment is due on the next April 1 and the rate of interest is 9% convertible semiannually.

2.12 A benefactor leaves an inheritance to four charities, A, B, C, and D. The total inheritance is a series of level payments at the end of each year forever. During the first \( n \) years A, B, and C share each payment equally. All payments after \( n \) years revert to D. If the present values of the shares of A, B, C and D are all equal, find \((1 + i)^n\).

2.13 A level perpetuity-immediate is to be shared by A, B, C and D. A receives the first \( n \) payments, B the second \( n \) payments, C the third \( n \) payments, and D all the payments thereafter. It is known that the ratio of the present value of C’s share to A’s share is .49. Find the ratio of the present value of B’s share to D’s share.

2.14 Compute \( a_{5.25} \) if \( i = 5\% \) using the following definitions:
   (a) Using formula
   \[
   a_{n+k} = a_n + v^{n+k} \left[ \frac{(1 + i)^k - 1}{i} \right].
   \]
   A payment at time 5.25 is \((1+.05)^{25}\).
   (b) A payment of .25 at time 5.25.
   (c) A payment of .25 at time 6.

2.15 A fund of $2000 is to be accumulated by \( n \) annual payments of $50, followed by \( n \) annual payments of $100, plus a smaller final payment made one year after the last regular payment. If the effective rate of interest is 4.5%, find \( n \) and the amount of the final irregular payment.

2.16 A borrower has the following two options for repaying a loan:
   (i) Sixty monthly payments of $100 at the end of each month.
   (ii) A single payment of $6000 at the end of each \( K \) months.
Interest is at the nominal annual rate of 12% convertible monthly. The two options have the same present value. Find $K$.

2.17 A fund of $17,000 is to be accumulated at the end of five years with payments at the end of each half-year. The first five payments are $1000 each, while the second five payments are $2000 each. Find the nominal rate of interest convertible semiannually earned on the fund.

2.18 A beneficiary receives a $10,000 life insurance benefit. If the beneficiary uses the proceeds to buy a 10-year annuity-immediate, the annual payout will be $1538. If a 20 year annuity-immediate is purchased, the annual payout will be $1072. Both calculations are based on an annual effective interest rate of $i$. Find $i$.

2.19 (a) Find the present value of an annuity-immediate which pays 1 at the end of each half-year for five years, if the rate of interest is 8% convertible semiannually for the first three years and 7% convertible semiannually for the last two years. (b) Find the present value of an annuity-immediate which pays 1 at the end of each half-year for five years, if the payments for the first three years are discounted at 8% convertible semiannually and the payments for the last two years are discount at 7% convertible semiannually. (c) Justify from general reasoning that the answer to (b) is larger than the answer to (a).

2.20 Find an expression for $a_{n}$ assuming each payment is valued at simple discount rate $d$.

Select 12 problems from the 20 problems above to finish!