4.1 If \( s(x) = 1 - x/100, 0 \leq x \leq 100 \), calculate

a. \( \mu(x) \)

b. \( F_X(x) \)

c. \( f_X(x) \)

d. \( \Pr(10 < X < 40) \).

4.2 Confirm that \( k\bar{q}_0 = -\Delta s(k) \), and that \( \sum_{k=0}^{\infty} k\bar{q}_0 = 1 \).

4.3 On the basis of Life Table in Chapter 4,

a. Compare the value of \( 5\bar{q}_0 \) and \( 5\bar{q}_5 \).

b. Evaluate the probability that (25) will die between ages 80 and 85.

4.4 Let the random variable \( K^*(x) = K(x), K(x) = 0, 1, 2, \ldots, n - 1 \)

\[ = n, K(x) = n, n + 1, \ldots \]

and denote \( \mathbb{E}[K^*(x)] \) by \( e_{x|n} \). This expectation is called a temporary curtate life expectancy. Show that

a. \( e_{x|n} = \sum_{k=0}^{n-1} k\bar{q}_x + n\bar{p}_x = \sum_{k=0}^{n} k\bar{p}_x \).

b. \( \text{Var}[K^*(x)] = \sum_{k=0}^{n-1} k^2\bar{q}_x + n^2\bar{p}_x - (e_{x|n})^2 = \sum_{k=0}^{n} (2k+1)k\bar{p}_x - (e_{x|n})^2 \).

4.5 If the random variable \( T(x) \) has d.f. given by

\[ F_T(t) = \begin{cases} \frac{t}{100-x} & 0 \leq t < 100 - x \\ 1 & t \geq 100 - x \end{cases} \]

Calculate

a. \( \mathbb{E}[T(x)] \),  b. \( \text{Var}[T(x)] \),  c. \( \text{median}[T(X)] \).

4.6 If \( \mu(x) = \mu \), a positive constant, for all \( x > 0 \), show that \( \bar{A}_x = \mu/(\mu + \delta) \).

4.7 Assume mortality is described by \( l_x = 100 - x \) for \( 0 \leq x \leq 100 \) and that the force of interest is \( \delta = 0.05 \).
a. Calculate $A_{40.25}^1$.

b. Determine the actuarial present value for a 25-year term insurance with benefit amount for death at time $t$, $b_t = e^{0.05t}$, for a person age 40 at policy issue.

4.8 The random variable $Z$ is the present-value random variable for a whole life insurance of unit amount payable at the moment of death and issued to $(x)$. If $\delta = 0.05$ and $\mu_x(t) = 0.01$:

a. Display the formula for the p.d.f of $Z$.

b. Calculate $\bar{A}_x = E[Z]$ and $\text{Var}(Z)$.

4.9 The random variable $Z$ is the present-value random variable for an $n$-year endowment insurance. Exhibit the d.f. of $Z$ in terms of the d.f. of $T$. 