Homework Assignment 2
(Due date: March. 8, 2010)

1. (10 marks) Prove that \((A \cup B) \cap (\overline{A} \cup C) = A \cap (B \cup C)\).

2. (30 marks) Draw a Venn diagram and shade the given set:
   (1) \((A \cup B) \cap (\overline{C} - A)\)
   (2) \(((C \cap A) - (B - \overline{A})) \cap C\)
   (3) \((B - \overline{C}) \cup ((B - \overline{A}) \cap (C \cup B))\)

3. (20 marks) Let \(f\) be a function from \(X\) to \(Y\). Define a relation \(R\) on \(X\) by
   \[xRy \quad \text{if} \quad f(x) = f(y).\]
   Show that \(R\) is an equivalence relation on \(X\).

4. (15 marks) Let \(S = x \in \mathbb{R} : x \geq -1\) and \(T = x \in \mathbb{R} : x \geq 0\), and define \(f(x) = \sqrt{x + 1}\) for \(x \in S\). Then \(f : S \rightarrow T\).
   (a) Show that \(f\) is one-to-one.
   (b) Show that \(f\) maps \(S\) onto \(T\).
   (c) Does \(f\) have an inverse? If so, find it.

5. (25 marks) Let \(R\) be a relation on a set \(A\). \(A = \{a, b, c, d\}\).
   (a) (5 marks) Write the relation \(R\), given by the matrix below, as a set of ordered pairs:
   \[
   \begin{pmatrix}
   1 & 0 & 1 & 0 \\
   0 & 0 & 0 & 0 \\
   0 & 0 & 1 & 0 \\
   1 & 1 & 1 & 1 \\
   \end{pmatrix}
   \]
   (b) (10 marks) Find out the reflexive closure and transitive closure of \(R\).
(c) (10 marks) If $S$ is a relation from $A$ to $B$, $B = \{1, 2, 3\}$ and the representing matrix of $S$ is
\[
\begin{pmatrix}
0 & 0 & 1 \\
1 & 0 & 1 \\
1 & 1 & 0 \\
0 & 1 & 0
\end{pmatrix},
\]
write the composite relation $S \circ R$. 