Homework Assignment III
(Due date: Mar. 26, 2010)

1. (20 marks) Let $P = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $Q = \{A, B, C, D, E\}$.
   (a) How many 4-element subsets of $P$ are there?
   (b) How many permutations, i.e., 5-permutations, of $Q$ are there?
   (c) How many license plates are there consisting of 3 letters from $Q$
       followed by 2 numbers from $P$? Repetition is allowed; for example,
       DAD88 is allowed. Explain how to get the results.

2. (20 marks) In how many ways can 8 people A,B,C,D,E,F,G and H be
   seated in a row if A and B must not sit next to each other and also D
   and E must not sit next to each other? For example, ABCDEFG and
   ACEDBFG are not allowed.
   Explain how to get the results.

3. (20 marks) Prove that there are $(2n - 1)(2n - 3)\cdots3.1$ ways to pick
   $n$ pairs from $2n$ distinct items.

4. (20 marks) How many permutations of the letters A,B,C,D,E,F,G con-
   tain
   a) contain the string BCD? (5 marks)
   b) contain the string CFGA? (5 marks)
   c) contain the string ABC and DE? (5 marks)
   d) contain AC but B,D can not be next to each other? (5 marks)

5. (20 marks) Suppose that there are piles of red, blue, and green balls
   and that each pile contains at least eight balls.
   (a) In how many ways can we select eight balls?
   (b) In how many ways can we select eight balls if we must have at
       least one ball of each color?