1. (20 marks) Let $p$ and $q$ be the propositions,

\[ p : \text{It is below freezing.} \]
\[ q : \text{It is snowing.} \]

Write these propositions using $p$ and $q$ and logical connectives.

(a) It is not below freezing and it is not snowing. (5 marks)
Sol: $\neg p \land \neg q$.

(b) If it is below freezing, it is also snowing. (5 marks)
Sol: $p \land q$.

(c) It is below freezing but not snowing. (5 marks)
Sol: $p \land \neg q$.

(d) That it is below freezing is necessary and sufficient for it to be snowing. (5 marks)
Sol: $p \leftrightarrow q$.

2. (20 marks) Show that $(p \rightarrow q) \land (q \rightarrow \gamma) \rightarrow (p \rightarrow \gamma)$ is a tautology.
Sol: Shown in the table below, the proposition is always true for any $(p, q, r)$, so is a tautology.

<table>
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<tr>
<th>$p$</th>
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<th>$(p \to q) \land (q \to r)$</th>
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3. (20 marks) In a standard deck of 52 playing cards, each of the 4 suits $\clubsuit, \diamondsuit, \heartsuit,$ and $\spadesuit$ contains 13 values including 2-10, J, Q, K and A. If five cards are selected, find the number of combinations of

(a) 5 hearts ($\heartsuit$); (8 marks)
Sol: $C_{13}^5 = 1287$.

(b) a pair of Ks and three other cards of different values. (12 marks)
Sol: $C_4^2 \times C_{48}^3 = 103776$.

4. (20 marks) Prove that $\overline{A \cap B \cap C} = \overline{A} \cup \overline{B} \cup \overline{C}$. Using set Identities
Sol: By De Morgan’s laws, it can be derived,
\[ A \cap B \cap C = A \cap B \cup C = A \cup B \cup C. \]

5. (20 marks) Let \( m, n \) be two positive integers. Consider two set
\( A = \{ a_1, a_2, \ldots, a_m \}, B = \{ b_1, b_2, \ldots, b_n \}. \) How many functions can be defined from \( A \) to \( B \) if
(a) no restriction when \( m \geq n \) (10 marks)?
Sol: \( n^m. \)
(b) the functions are bijection functions when \( m = n \) ? (10 marks)
Sol: \( m! \) (or \( n! \)).