

Distinguished Lecture Series

Crouzeix's Conjecture



Professor Michael Overton

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Biography:

Michael L. Overton is Silver Professor of Computer Science and Mathematics at the Courant Institute of Mathematical Sciences, New York University. He received his Ph.D. in Computer Science from Stanford University in 1979. He is a fellow of SIAM (Society for Industrial and Applied Mathematics) and of the IMA (Institute of Mathematics and its Applications, UK). He served on the Council and Board of Trustees of SIAM from 1991 to 2005, including a term as Chair of the Board from 2004 to 2005. He served as Editor-in-Chief of SIAM Journal on Optimization from 1995 to 1999 and of the IMA Journal of Numerical Analysis from 2007 to 2008, and was the Editor-in-Chief of the MPS (Mathematical Programming Society)-SIAM joint book series from 2003 to 2007. He is currently an editor of the IMA Journal of Numerical Analysis, Foundations of Computational Mathematics, Numerische Mathematik and Calcolo. His research interests are at the interface of optimization and linear algebra, especially nonsmooth optimization problems involving eigenvalues, pseudospectra, stability and robust control. He is the author of "Numerical Computing with IEEE Floating Point Arithmetic" (SIAM, 2001).

Date: 19 October 2022 (Wednesday)

Time: 10:00-11:00 a.m. GMT+8 (Hong Kong Time)

Venue: Online via Zoom (Meeting ID: 929 8218 5199)

Abstract

Crouzeix's conjecture is among the most intriguing developments in matrix theory in recent years. Made in 2004 by Michel Crouzeix, it postulates that, for any polynomial p and any matrix A , $\|p(A)\| \leq 2 \max(|p(z)|: z \in W(A))$, where the norm is the 2-norm and $W(A)$ is the field of values (numerical range) of A , that is the set of points attained by v^*Av for some vector v of unit length. Crouzeix proved in 2007 that the inequality above holds if 2 is replaced by 11.08, and in 2016 this was greatly improved by Palencia, replacing 2 by $1+\sqrt{2}$. Furthermore, it is known that the conjecture holds in a number of special cases, including $n=2$. We use nonsmooth optimization to investigate the conjecture numerically by locally minimizing the "Crouzeix ratio", defined as the quotient with numerator the right-hand side and denominator the left-hand side of the conjectured inequality. We also present local nonsmooth variational analysis of the Crouzeix ratio at conjectured global minimizers. All our results strongly support the truth of Crouzeix's conjecture.

This is joint work with Anne Greenbaum and Adrian Lewis.

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