香港浸會大學
HONG KONG BAPTIST UNIVERSITY

50th Anniversary

香港浸會大學理學院
HKBU Faculty of Science

Department of Mathematics

50th Anniversary Lecture Series

On a Conjecture of C. Sundberg: A Numerical Investigation

Professor Roland Glowinski

Cullen Professor of Mathematics, University of Houston
Member of the French National Academy of Sciences
Member of the Academia Europaea
Member of the French National Academy of Technology
Seymour Cray Prize-France (1988)
Grand Prix Marcel Dassault (1996)
SIAM Theodore Von Kármán Prize (2004)

Date: **24 May 2011 (Tuesday)**

Time: **4:15 - 5:15 pm** (Preceded by Reception at 3:45 pm)

Venue: **FSC1217, Fong Shu Chuen Building,**
Ho Sin Hang Campus,
Hong Kong Baptist University

Abstract

Carl Sundberg from University of Tennessee-Knoxville conjectured that

$$\sup_{v\in S}\frac{\displaystyle\int_0^1 \frac{|v'|^4}{v^6}dx}{1+\displaystyle\int_0^1 |v''|^2dx} < +\infty, \qquad \text{(CSI)}$$

where

$$S=\{v|v\in H^2(0,1),\ v(0)=v(1),\ v'(0)=v'(1),\ v\geq 1\}.$$

Our goal is to use a computational approach to investigate if (CSI) holds and to compute the related supremum, assuming it is finite. To do so, we observe that the supremum in (CSI) is equal to:

$$\sup_{\alpha\geq 0}\frac{F(\alpha)}{1+\alpha},$$

where

$$F(\alpha)=\sup_{v\in S_\alpha}\int_0^1\frac{|v'|^4}{v^6}dx$$

and

$$S_\alpha=\left\{v|v\in S,\ \int_0^1 |v''|^2dx=\alpha\right\}.$$

The strategy we advocate at the moment is a pretty crude one, namely, tabulate the function $\alpha\rightarrow\frac{F(\alpha)}{1+\alpha}$ in order to get information on the boundedness of the supremum in (CSI). In our lecture, we will discuss the numerical computation of $F(\alpha)$, the associated problem of Calculus of Variations being solved by a methodology combining a finite difference discretization and an augmented Lagrangian algorithm associated with the following three families of linear constraints $v-q_0=0$, $v'-q_1=0$ and $v''-q_2=0$.

The results of numerical experiments (with α in the range $[0,10^6]$) will be presented; we will discuss the conclusions we can draw from them concerning the veracity of (CSI).

✦ ✦ ✦ **All are welcome** ✦ ✦ ✦

For enquires please contact Ms. Claudia Chui, 3411 2348.

http://www.math.hkbu.edu.hk/