Application of ADMM to the Solution of a Nonconvex Variational Problem

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In relation with the investigations of ***B. Dacorogna*** and collaborators (ref. [1]) on the ***equivalence of differential forms***, an important problem in Geometry and Analysis, we discuss the numerical solution of the following non-convex variational problem:

$$\begin{cases} \mathbf{u} \in \mathbf{E}, \\ J_\varepsilon(\mathbf{u}) \leq J_\varepsilon(\mathbf{v}), \ \forall \mathbf{v} \in \mathbf{E}, \end{cases} \tag{P$_\varepsilon$}$$

where $\varepsilon > 0$.

$$\mathbf{E} = \{\mathbf{v} | \mathbf{v} \in (H^2(\Omega))^2, \ \mathbf{v}(\mathbf{x}) = \mathbf{x} \text{ on } \partial\Omega, \ \det \boldsymbol{\nabla} \mathbf{v} = f (> 0)\},$$
$$J_\varepsilon(\mathbf{v}) = \frac{\varepsilon}{2} \int_\Omega |\nabla^2 \mathbf{v}|^2 d\mathbf{x} + \frac{1}{2} \int_\Omega |\nabla \mathbf{v} - \mathbf{I}|^2 d\mathbf{x}.$$

In order to solve (P$_\varepsilon$) we advocate an algorithm of the ***ADMM*** type, closely related to those we employed in [2] to solve nonlinear variational problems from ***Finite Elasticity***.

References

[1] CSATO, G., B. DACOROGNA & O. KNEUSS, *The Pullback Equation for Differential Forms*, Birkhauser, New York, NY, 2012.

[2] GLOWINSKI R. & P. LE TALLEC, *Augmented Lagrangians and Operator-Splitting Methods in Nonlinear Mechanics*, SIAM, Philadelphia, PA, 1989.