Extrapolation Cascadic Multigrid Method

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A new extrapolation cascadic multi-grid method (EXCMG) in solving the FE-system of equations is proposed.

EXCMG= new extrapolation + quadratic interpolation + CG.

The experiments and analysis show that EXCMG is convergent and superconvergent in both K-norm and discrete L^2-norms, when the solution is smooth or nonsmooth, $u \in H^3, H^2$.

To exhibit these new extrapoltion formulas, consider one-dimensional coarse element $I_j = (x_j, x_{j+1})$, and the linear FE-solutions in double grids given,

$$Z_0: \ \{U_j^0,\ U_{j+1}^0\}; \qquad Z_1: \ \{U_j^1,\ U_{j+1/2}^1,\ U_{j+1}^1\}.$$

Using new extrapolation formulas and quadratic interpolation, we can provide five better nodal values $W^2 = F(U^1, U^0)$ on Z_2 as follows:

$$\begin{aligned}
W_k^2 &= U_k^1 + (U_k^1 - U_k^0)/4, \qquad k = j,\ j+1,\\
W_{j+1/2}^2 &= U_{j+1/2}^1 + \tfrac{1}{8}\{(U^1 - U^0)_j + (U^1 - U^0)_{j+1}\},\\
W_{j+1/4}^2 &= \{(9U_j^1 + 12U_{j+1/2}^1 - U_{j+1}^1) - (3U_j^0 + U_{j+1}^0)\}/16,\\
W_{j+3/4}^2 &= \{(9U_{j+1}^1 + 12U_{j+1/2}^1 - U_j^1) - (3U_{j+1}^0 + U_j^0)\}/16.
\end{aligned}$$

We have used CG-iteration on Z_l with the iteration times $m_l = m_L\beta^{L-l}$, $\beta = 4$, $l = 2, 3, ..., L$. Fianlly, to improve the accuracy of the final iteration solution u_L^*, we shall accepted the classical extrapolation and superconvergence techniques for the function value and gradient, respectively.