The Blessing of Dimensionality

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Numerical problems in many dimensions are usually difficult or impossible to solve because of the curse of dimensionality, i.e. the exponential growth of the complexity in the number of axes. In this regard the classical tensor decompositions are not very helful in multidimensional numerical analysis. A kind of break-through has been started in 2009 due to the new algorithms of tensor-train decompositions (chiefly the TT-SVD rounding and TT-cross interpolation algorithms). The complexity of TT is linear in the number of axes. Moreover, the tensor-train decomposition is now coupled with the idea of introducing as many fictitious axes as possible (quantization), which means that we now welcome the increase of dimensionality and use it to derive more efficient algorithms, e.g. with the logarithmic complexity in the data size. A recent finding is a connection of the TT with wavelets and the possibility to use it for the construction of new filter banks by purely algebraic methods, this leads to algebraic wavelet transforms that are well-adapted to signals and justly called WTT (wavelet-tensor-train) tranforms.

References

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