Local Oscillations in Finite Difference Solutions of Hyperbolic Conservation Laws

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It was generally expected that monotone schemes are oscillation-free for hyperbolic conservation laws. However, recently local oscillations were observed and usually understood to be caused by relative phase errors. In order to further explain this, we first investigate the discretization of initial data that trigger the chequerboard mode, the highest frequency mode. Then we proceed to use the discrete Fourier analysis and the modified equation analysis to distinguish the dissipative and dispersive effects of numerical schemes for low frequency and high frequency modes, respectively. It is shown that the relative phase error is of order $\mathcal{O}(1)$ for the high frequency modes $u_j^n = \lambda_k^n e^{i\xi j}$, $\xi \approx \pi$, but of order $\mathcal{O}(\xi^2)$ for low frequency modes ($\xi \approx 0$). In order to avoid numerical oscillations, the relative phase errors should be offset by numerical dissipation of at least the same order. Numerical damping, i.e. the zero order term in the corresponding modified equation, is important to dissipate the oscillations caused by the relative phase errors of high frequency modes. This is in contrast to the role of numerical viscosity, the second order term, which is the lowest order term usually present to suppress the relative phase errors of low frequency modes.

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