Talk: Well-posedness of the fractional Benjamin-Ono equation and dispersive limit behavior between some dispersive equations

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Abstract:

The dispersive perturbation of the inviscid Burgers equation

$$\partial_t u + D_x^{\alpha-1}\mathcal{H}\,\partial_x u + \partial_x(u^2) = 0$$

is considered. It follows that it is locally well-posed in $H^{1/2}$ with $3/2 < \alpha \le 2$. Moreover, well-posedness of Camassa-Holm equation

$$u_t + \alpha u_x - u_{txx} + uu_x = 2u_x u_{xx} + uu_{xxx}\,,\ \ \alpha > 0$$

Is considered. Local well-posedness of Camassa-Holm equation in endpoint space $H^{3/2}$ is obtained under the conditions for initial data that $u_0 - u_{0xx}$ is nonnegative and belongs to L^1.

Moreover, we also consider the dispersive limit behavior from KdV-Benjamin-Ono equation to Benjamin-Ono equation, and show that solution of the Cauchy problem for the KdV-Benjamin-Ono equation converges to the solution of Cauchy problem for the Benjamin-Ono equation in the natural space $C([0,T];H^{1/2})$ for any $T > 0$ as KdV term tends to zero.

Finally, we consider the dispersive limit behavior of finite-depth-fluid equation,

$$u_t - \mathcal{G}_\delta(u_{xx}) + uu_x = 0.$$

We show that the solution of the finite-depth-fluid equation converge in $C(0,T;H^{1/2})$ to the solution of the Benjamin-Ono equation as the depth δ tends to infinity; that the solution of the finite-depth-fluid equation converge in $C(0,T;H^{1/2})$ to the solution of the KdV equation as the depth δ tends to 0. If $H^{1/2}$ is replaced by H^1, then $T = \infty$. The Cauchy problem of finite-depth-fluid equation is locally well-posed in $H^{1/2}$, and it is globally well-posed in H^1.