創意研究院
普及數學公開講座
網絡中的數學
The Mathematics of Networks

吳國寶 Michael Ng

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HONG KONG BAPTIST UNIVERSITY
Department of Mathematics

普及數學公開講座
Public Lecture on Mathematics

美國最高法院：資料與數據
U.S. Supreme Court: Facts and Figures

陸大章教授
香港浸會大學副校長(學術)

Professor Franklin Luk
HKBU Vice-President (Academic)

Wofoo Public Lecture in Mathematics

現代數學與現代藝術

Prof. Tianxin Cai
蔡天新教授

Zhejiang University
浙江大學

香港浸會大學
HONG KONG BAPTIST UNIVERSITY
Department of Mathematics

普及數學公開講座
Public Lecture on Mathematics

布拉格天文鐘的數學原理是什麼呢?
What mathematics is hidden behind the astronomical clock of Prague?

Dr. Alena Solcova

Department of Mathematics
Czech Technical University, Prague

Date: 22 February 2008 (Friday)
Time: 4:30 pm (Preceded by Reception at 4:00 pm)
Venue: University Chapel, Ho Sin Hang Campus

Public Lecture on Mathematics

Computational Thinking: A Necessary Subject in Education

Professor Walter Gander

岗沃德特

ETH (Zürich, Switzerland) and
Hong Kong Baptist University

香港浸會大學
HONG KONG BAPTIST UNIVERSITY
Department of Mathematics

普及數學公開講座
Public Lecture on Mathematics

從機器學習到人類創新
From Machine Learning to Human Innovation

Professor Edward Chang

Director of Research, Google China
Professor of University of California, Santa Barbara

50th Anniversary Lecture Series

一個幾何公式的故事 ——
從扭轉、絞擰到基因、藤蔓、太陽爆發

Professor Boju Jiang

姜伯駒教授

Peking University

北京大學

Science Distinguished Lecture Series

Mathematics is Around Us

Professor Zhi-Ming Ma

Professor, Institute of Applied Mathematics, AMSS, CAS
Fellow of Chinese Academy of Sciences
Fellow of the Academy of Sciences for the Developing World
President, Chinese Mathematical Society, 2008-2011
Vice President, Executive Committee of International Math Union, 2007-2010
Medallion Lecture, World Congress in Probability and Statistics, 2007
Hua Loo-Keng Mathematics Prize, 2005
S.S. Chern Mathematics Prize, 1995
Invited speaker, International Congress of Mathematicians, 1994
Chinese National Natural Sciences Prize, 1993
Max-Plank Research Award, 1992

甚麼是大數據

Mr. Herbert Chia

Vice President of Data Technology and Product Departmen
and Head of Data Committee at Alibaba Group

Lecture on Mathematics
普及數學公開講座

微積分和無限的文化

林群

中國科學院院士
第三世界科學院院士
中國科學院數學與系統科學研究院教授

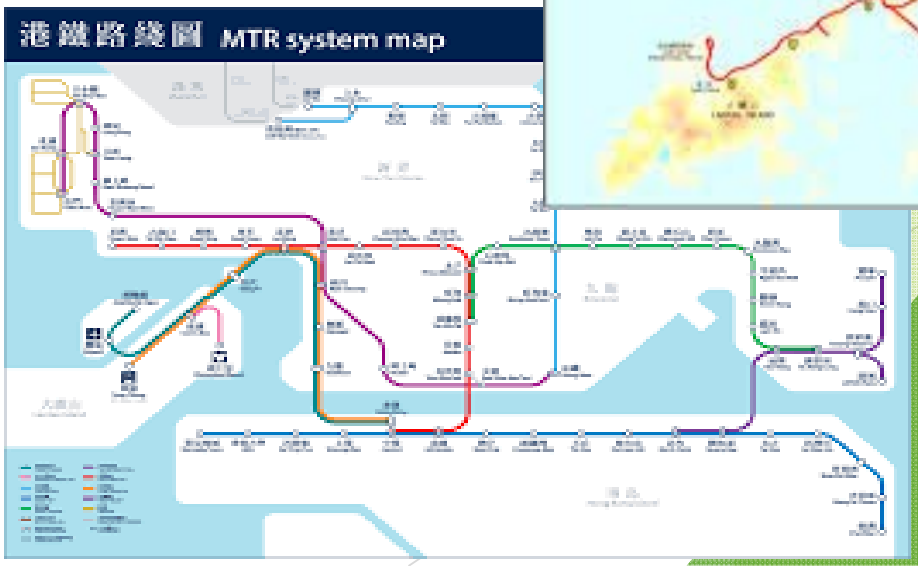
網絡中的數學

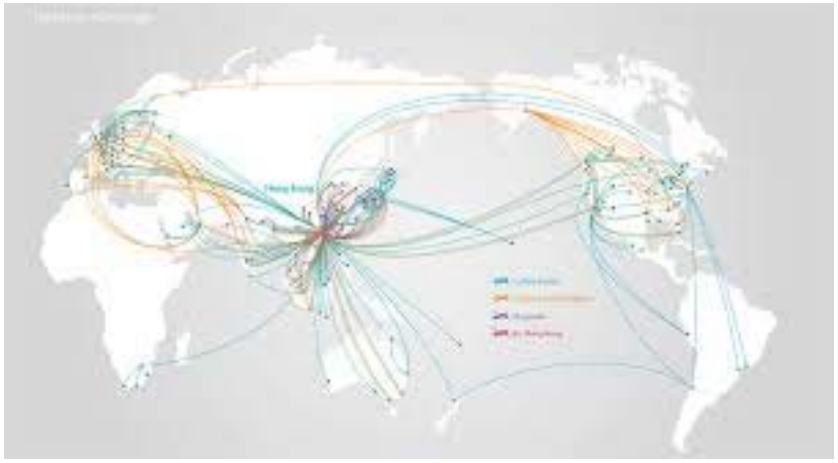
本演講共分三部份:

- 1. 介紹基本圖論，了解網絡數學。
- 2. 介紹谷歌搜索引擎，及其相關的數學理論。
- 3. 簡介網絡排名方法在生物，工程，科學及經濟等的應用。

網絡 (Networks)

- Mobile Phone Network
- Road Network
- MTR Network
- Airline Network





Social Networks

facebook

Connect with friends and the world around you on Facebook.

Home About

LinkedIn Corporation

Company

LinkedIn /ˌlɪŋkt.ˈɪn/ is a business-oriented social networking service. Founded in December 2002 and launched on May 5, 2003, it is mainly used for professional networking. Wikipedia

Welcome to Twitter.

Connect with your friends — and other fascinating

WhatsApp Web

Use WhatsApp on your phone to scan the code

Keep me signed in

To reduce mobile data usage, connect your phone to Wi-Fi

Instagram

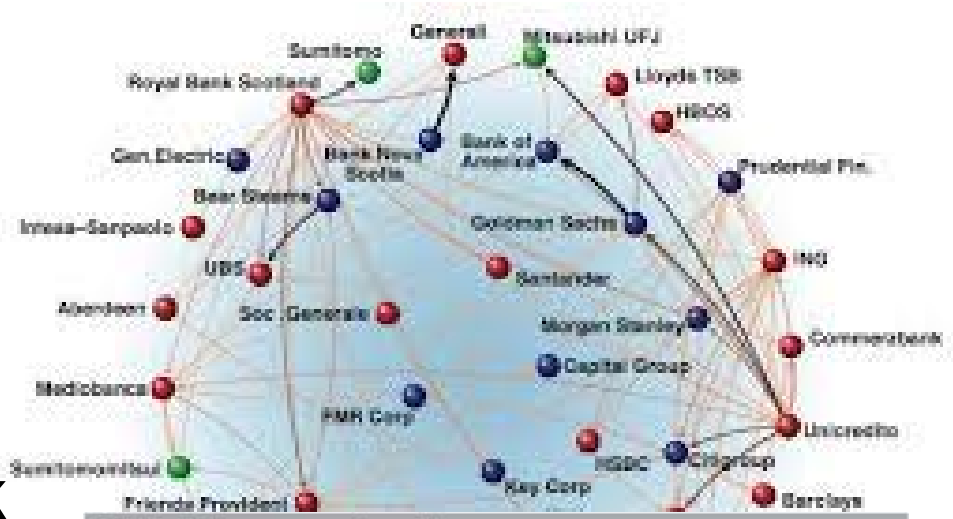
Username

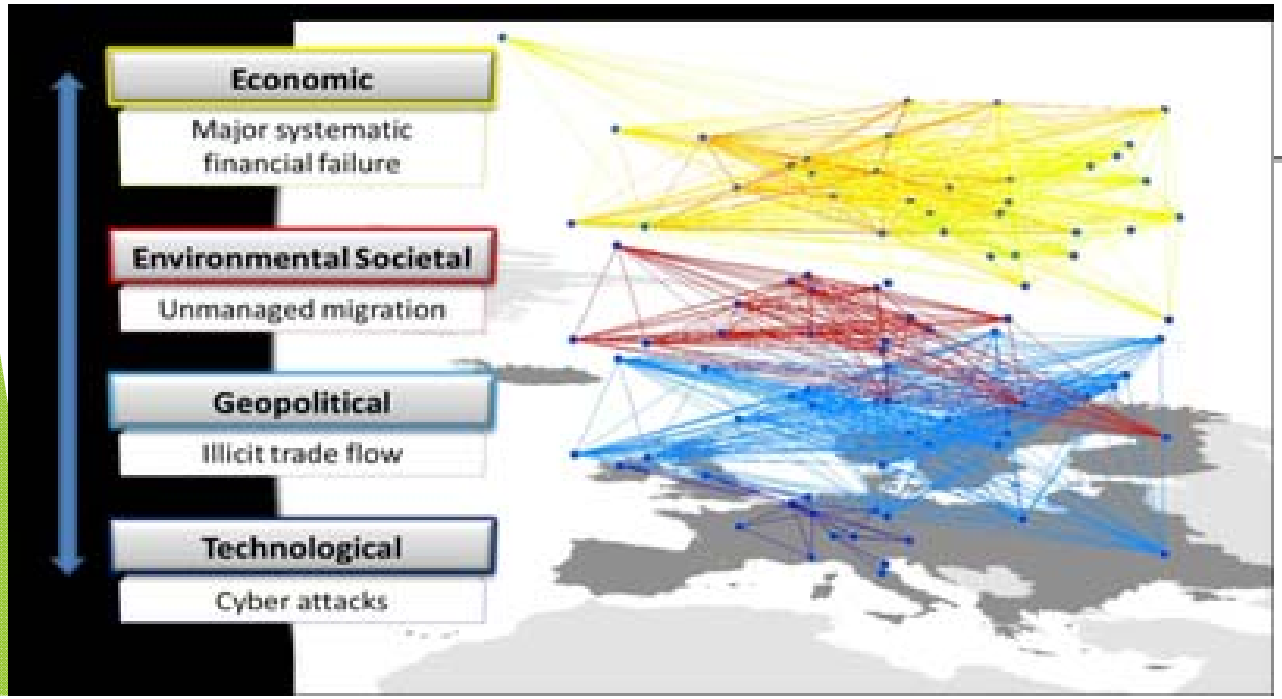
Password Forgot?

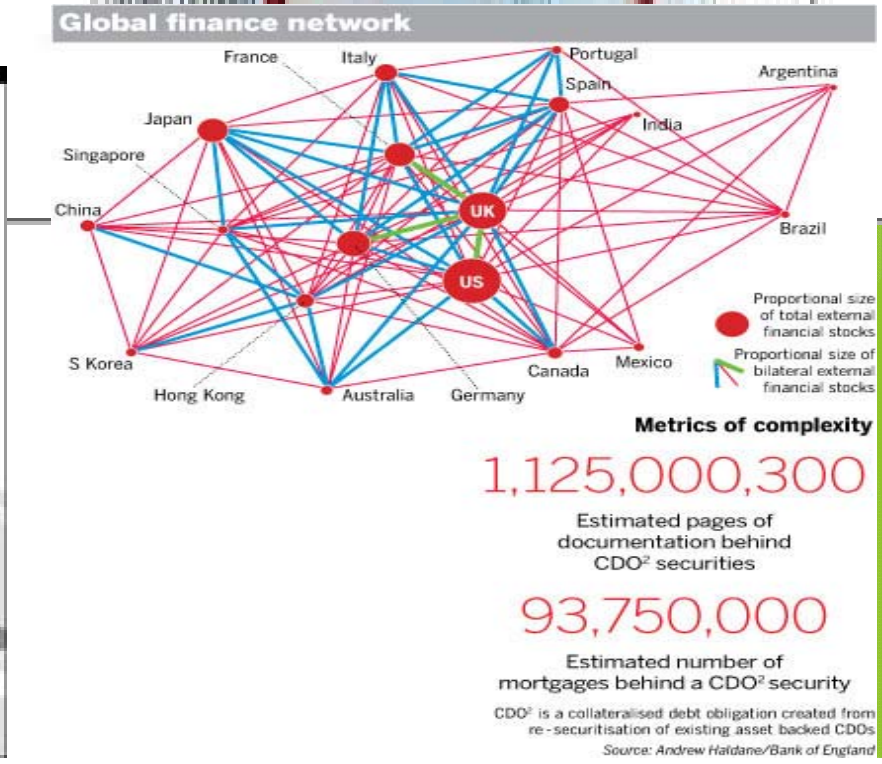
Log in

網絡 (Networks)

- Company Network
- Finance Network
- Social and Economical Network

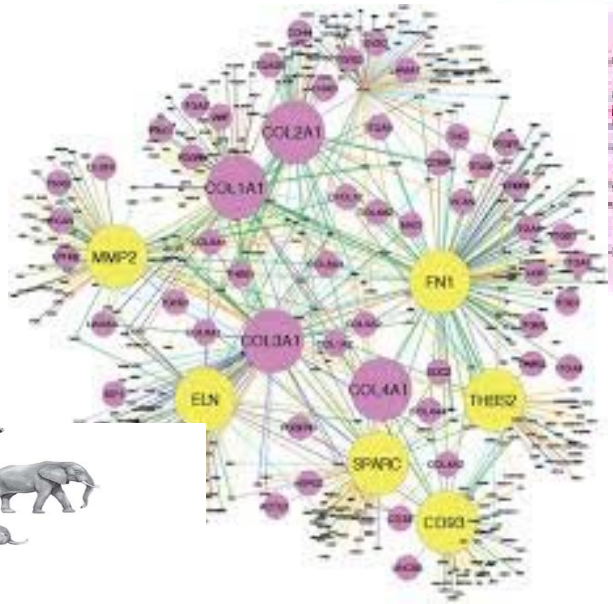


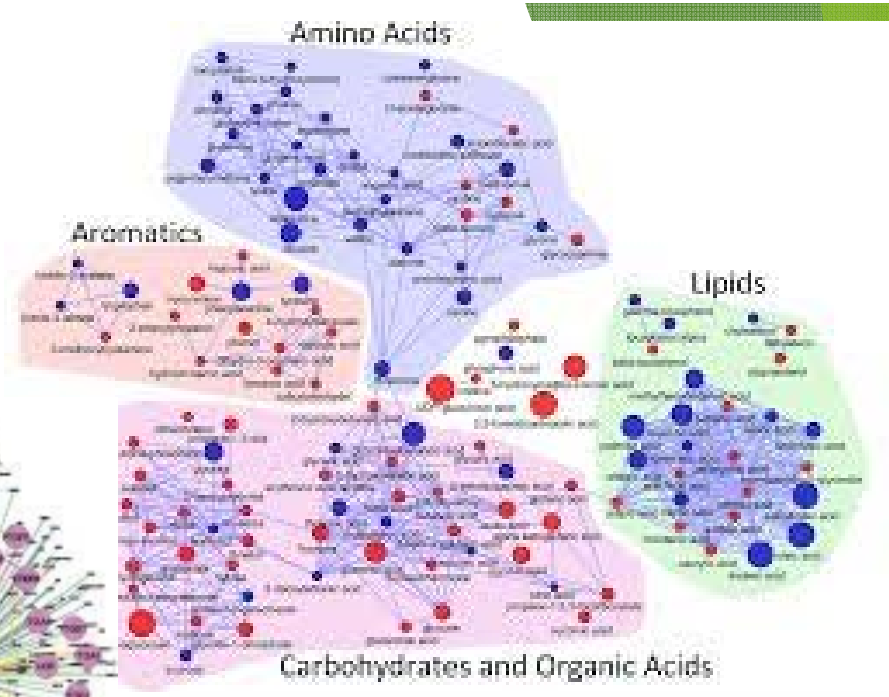


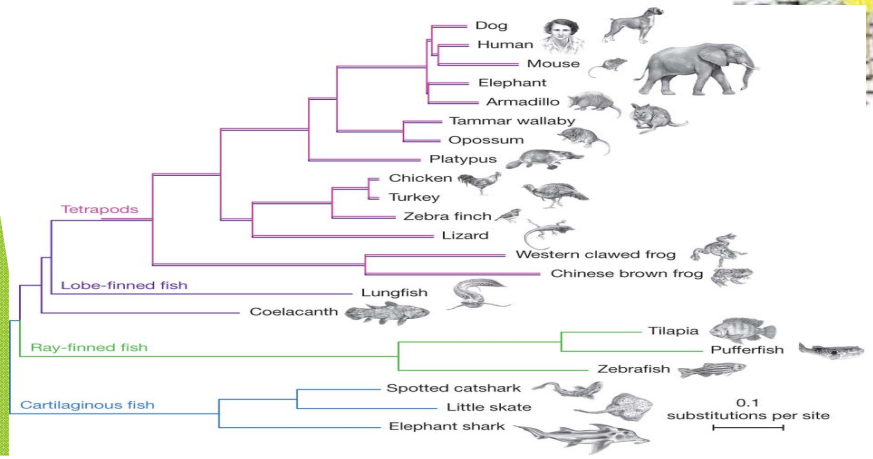


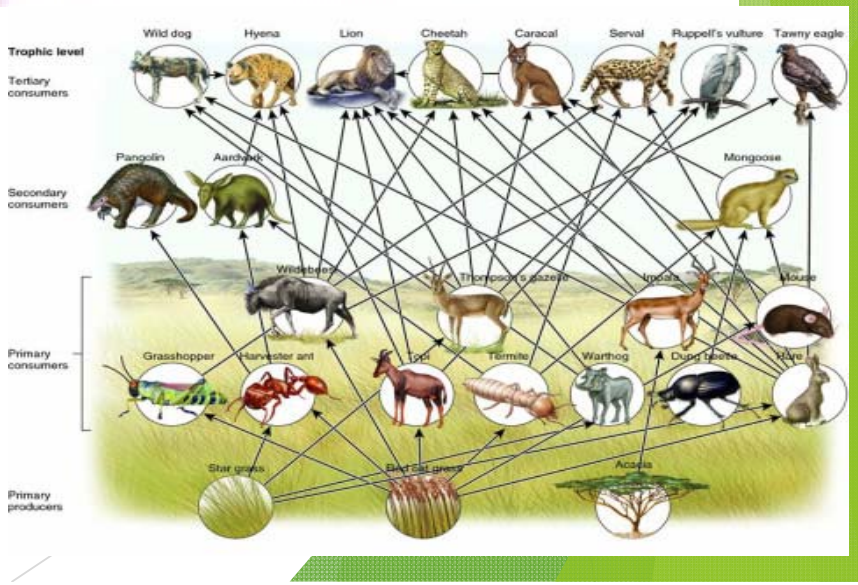
網絡 (Networks)

- Molecular Networks
- Gene Networks
- Food Tree
- Phylogenetic
- Tree



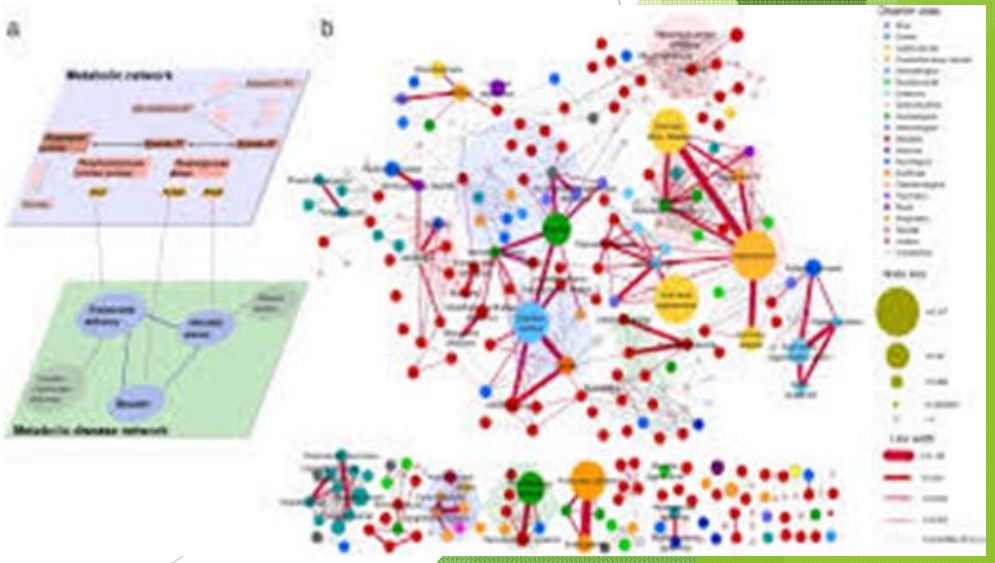






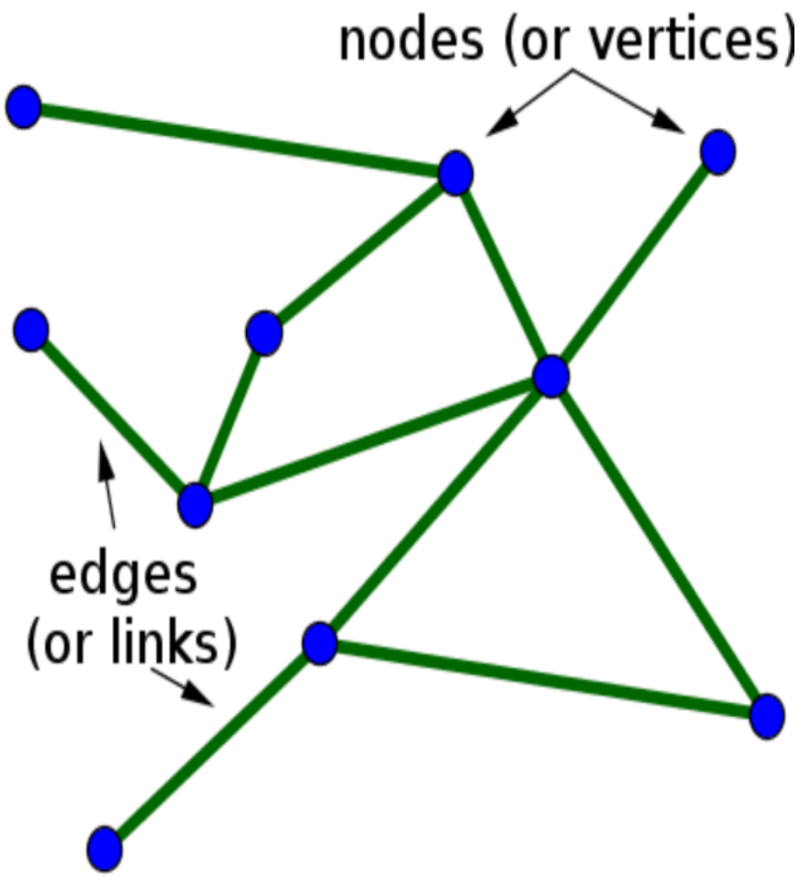
網絡 (Networks)

- Visualization
- Interpret and analyze ?
- 1 billion webpages
- 1.59 billion Facebook users (4.4 millions at HK)
- 1 billion Whatapps users
- 14 millions mobile users at Hong Kong

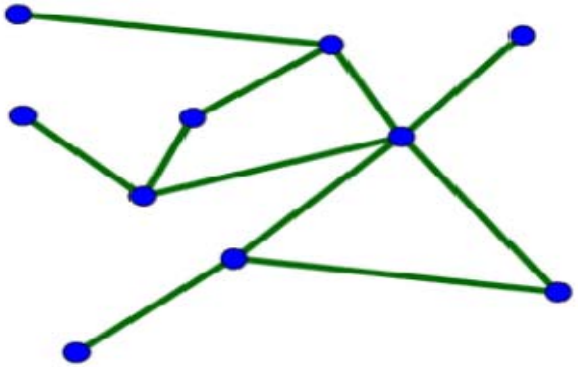


圖論 (Graph Theory)

- A network is simply a collection of connected objects.
- The objects as nodes or vertices, and usually draw them as points.
- The connections between the nodes as edges, and usually draw them as lines between points.
- In mathematics, networks are often referred to as graphs.

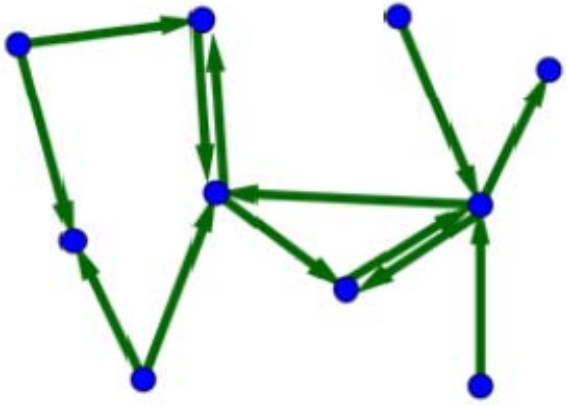


圖論 (Graph Theory)



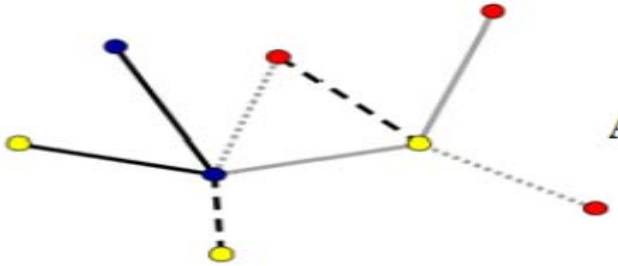
An undirected network.

road, mtr networks



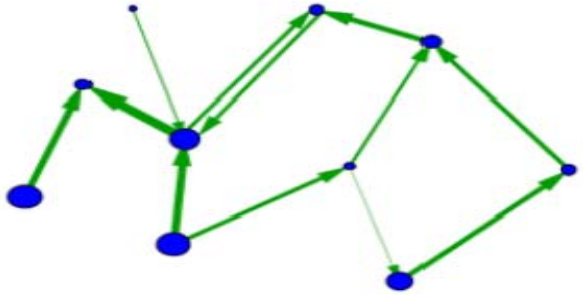
A directed network.

WWW, food networks



An undirected network where the nodes and edges have different types, as indicated by their colors and line styles.

Metabolic network, enzymes (behaviour and interactions)

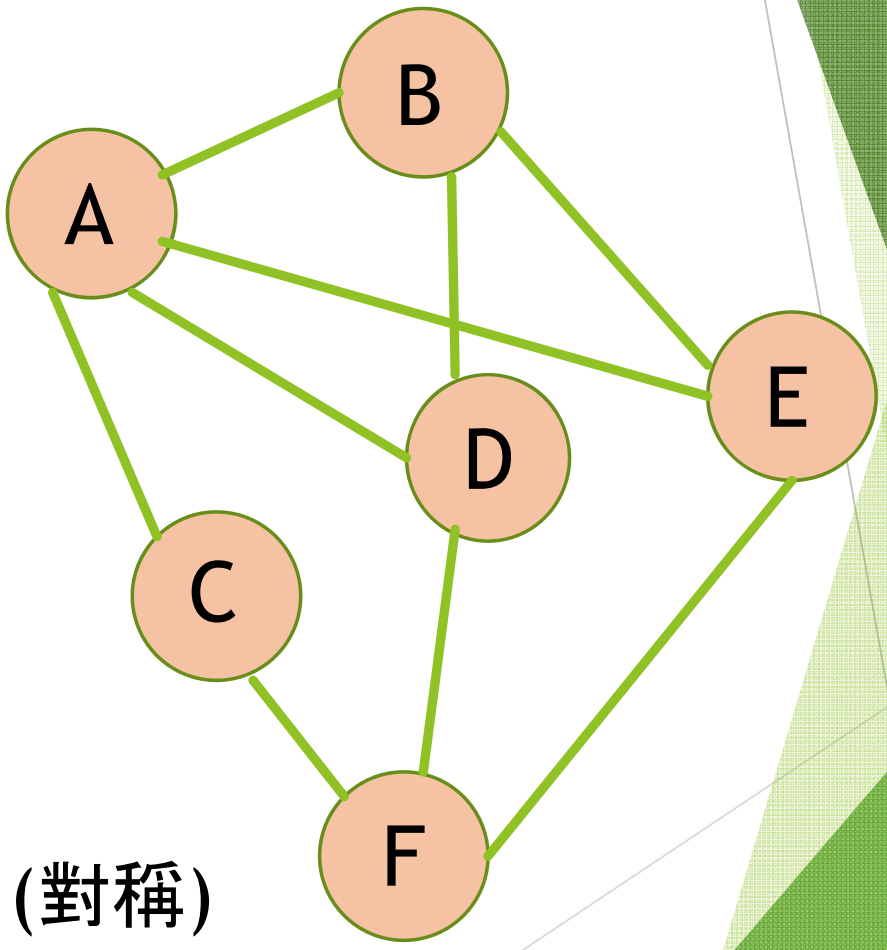


A directed network where the edges and nodes have different weights, as indicated by their sizes.

Finance network

鄰接矩陣 (Adjacency Matrix)

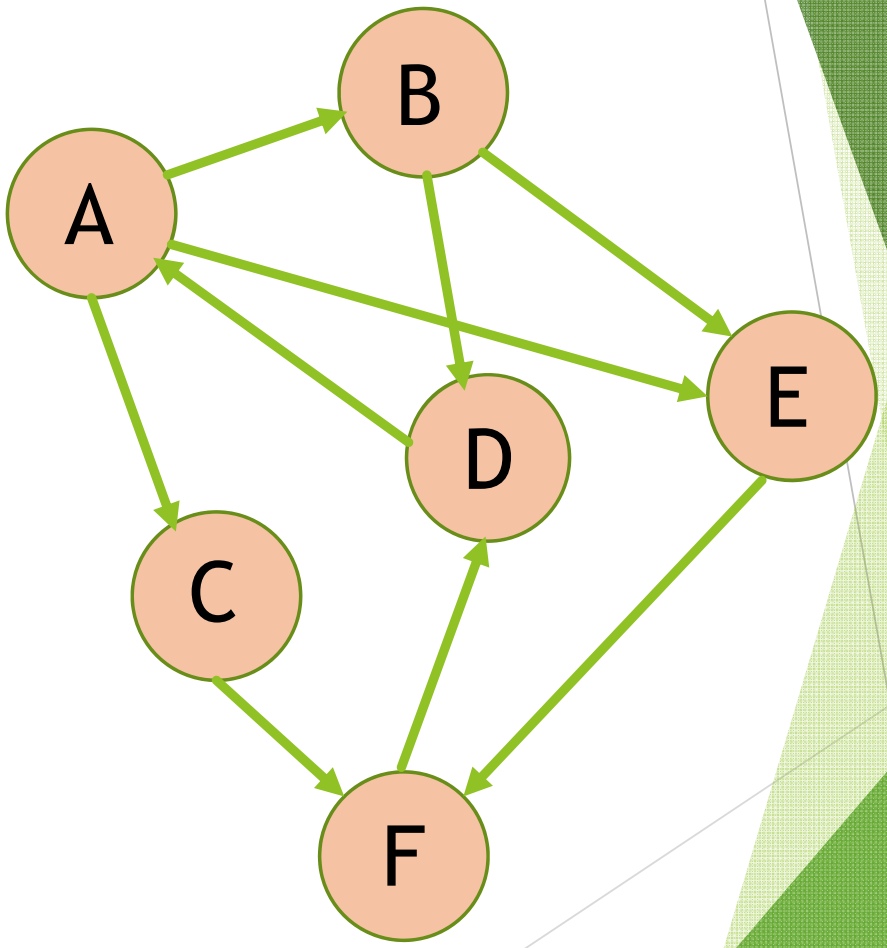
	A	B	C	D	E	F
A	---	1	1	1	1	0
B	1	---	0	1	1	0
C	1	0	---	0	0	1
D	1	1	0	---	0	1
E	1	1	0	0	---	1
F	0	0	1	1	1	---



6 nodes; 6 rows (列) and
6 columns (行); symmetric (對稱)

鄰接矩陣 (Adjacency Matrix)

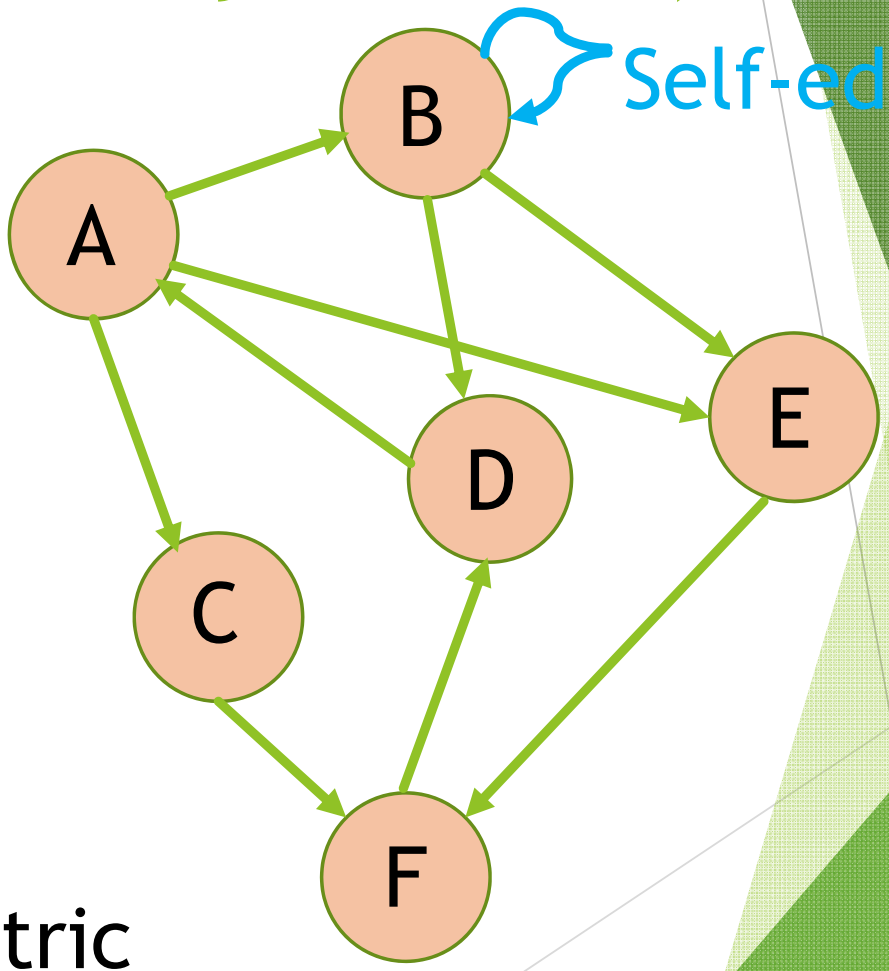
	A	B	C	D	E	F
A	---	0	0	1	0	0
B	1	---	0	0	0	0
C	1	0	---	0	0	0
D	0	1	0	---	0	1
E	1	1	0	0	---	0
F	0	0	1	0	1	---



6 nodes; 6 rows (列) and
6 columns (行);
not symmetric(非對稱)

鄰接矩陣 (Adjacency Matrix)

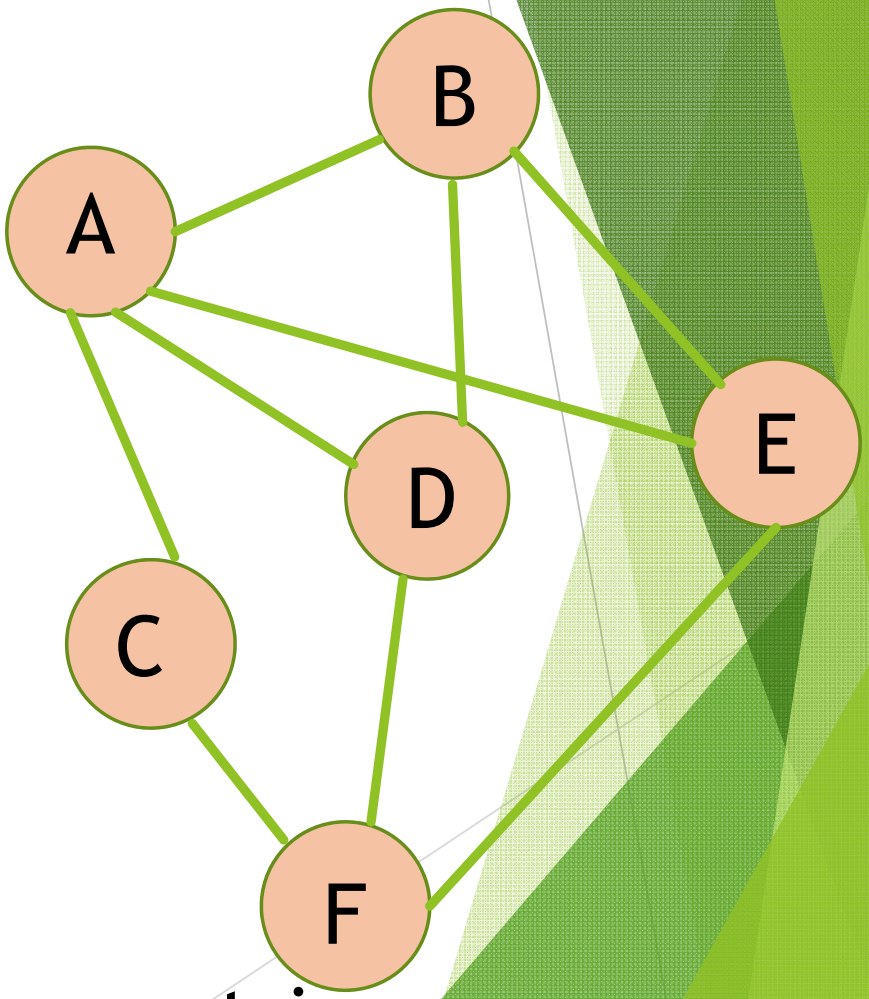
	A	B	C	D	E	F
A	---	0	0	1	0	0
B	1	1	0	0	0	0
C	1	0	---	0	0	0
D	0	1	0	---	0	1
E	1	1	0	0	---	0
F	0	0	1	0	1	---



6 nodes; 6 rows and
6 columns; not symmetric

Degree

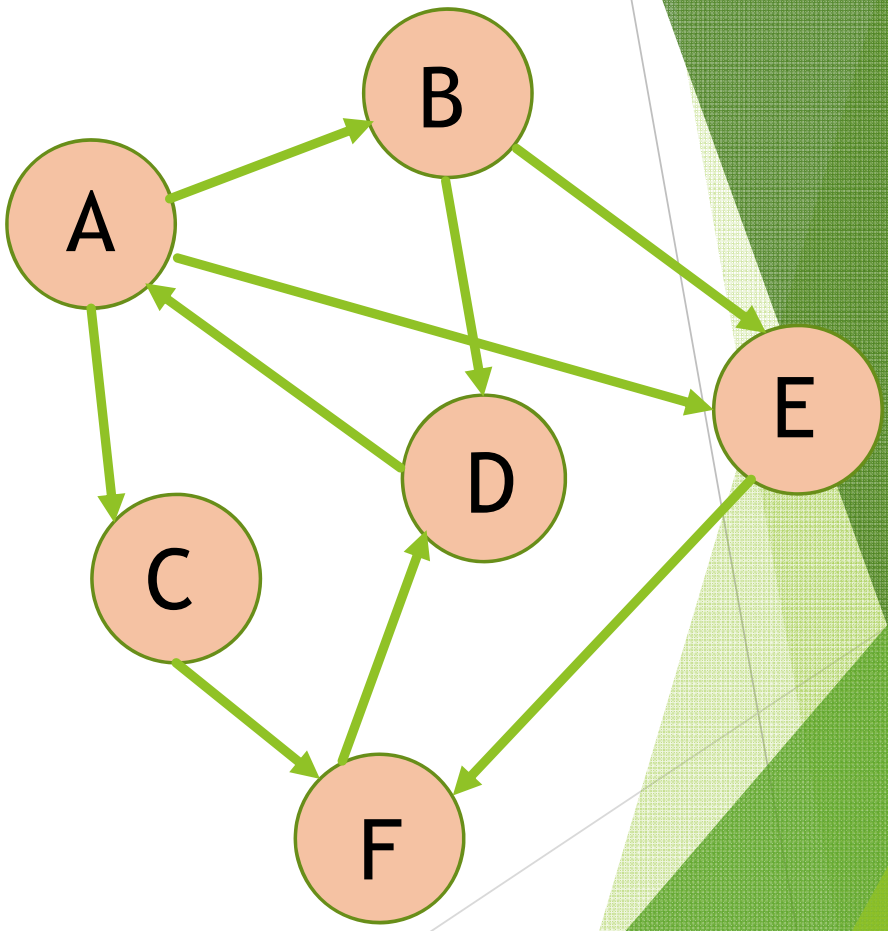
	A	B	C	D	E	F	In Degree
A	---	1	1	1	1	0	4
B	1	---	0	1	1	0	3
C	1	0	---	0	0	1	2
D	1	1	0	---	0	1	3
E	1	1	0	0	---	1	3
F	0	0	1	1	1	---	3
Out Degree	4	3	2	3	3	3	



6 nodes; 6 rows and 6 columns; symmetric

In and Out Degrees

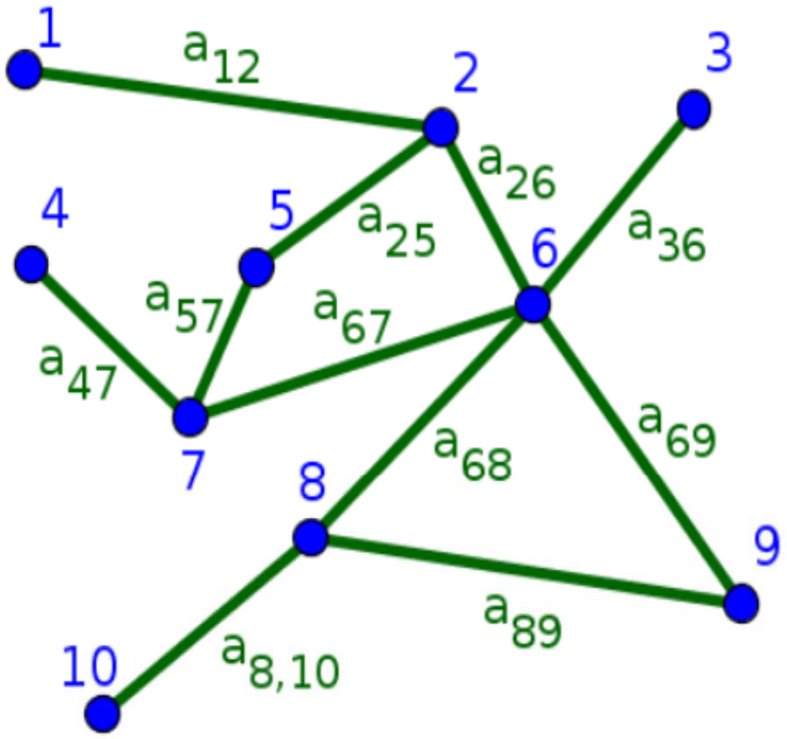
	A	B	C	D	E	F	In Degree
A	---	0	0	1	0	0	1
B	1	---	0	0	0	0	1
C	1	0	---	0	0	0	1
D	0	1	0	---	0	1	2
E	1	1	0	0	---	0	2
F	0	0	1	0	1	---	2
Out Degree	3	2	1	1	1	1	

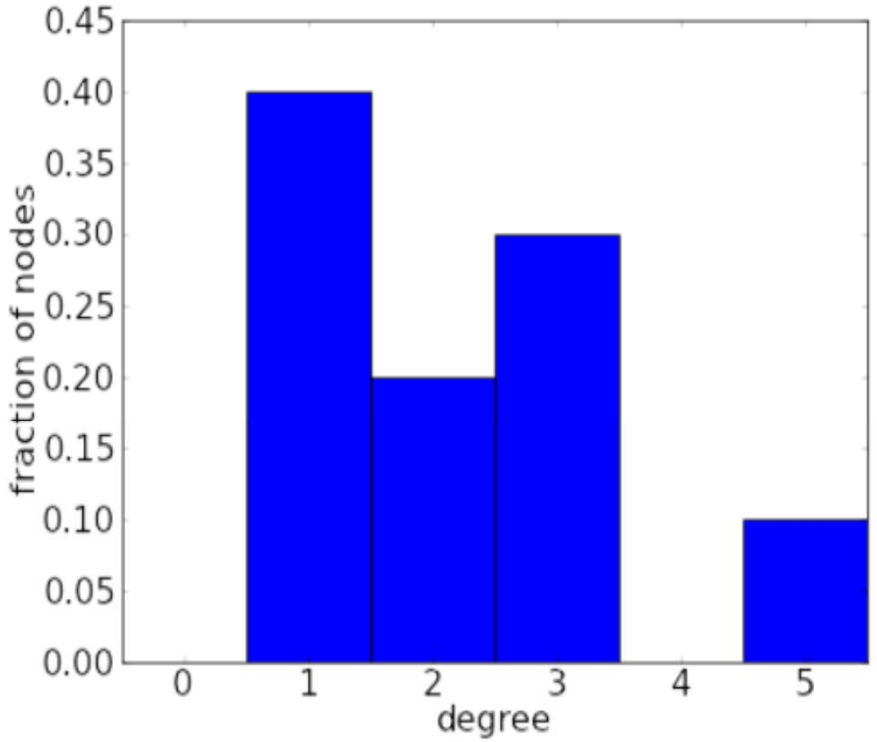


6 nodes; 6 rows and
6 columns; not symmetric

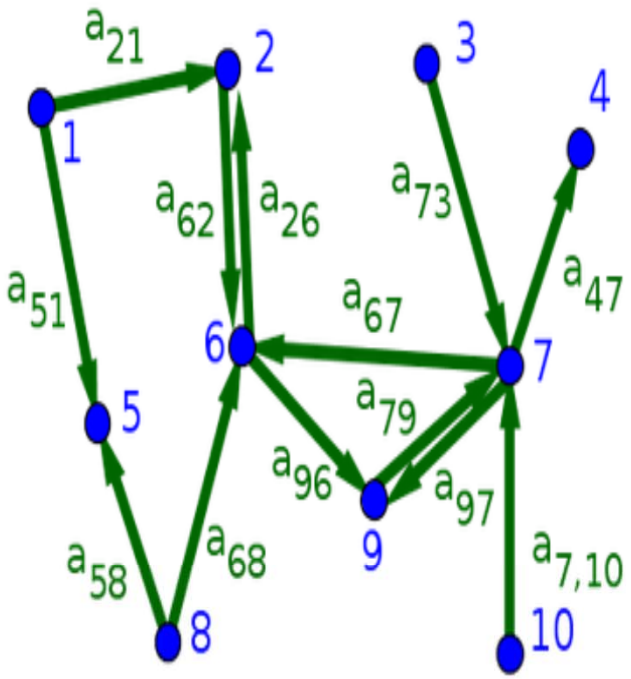
Degree Distribution (Histogram)

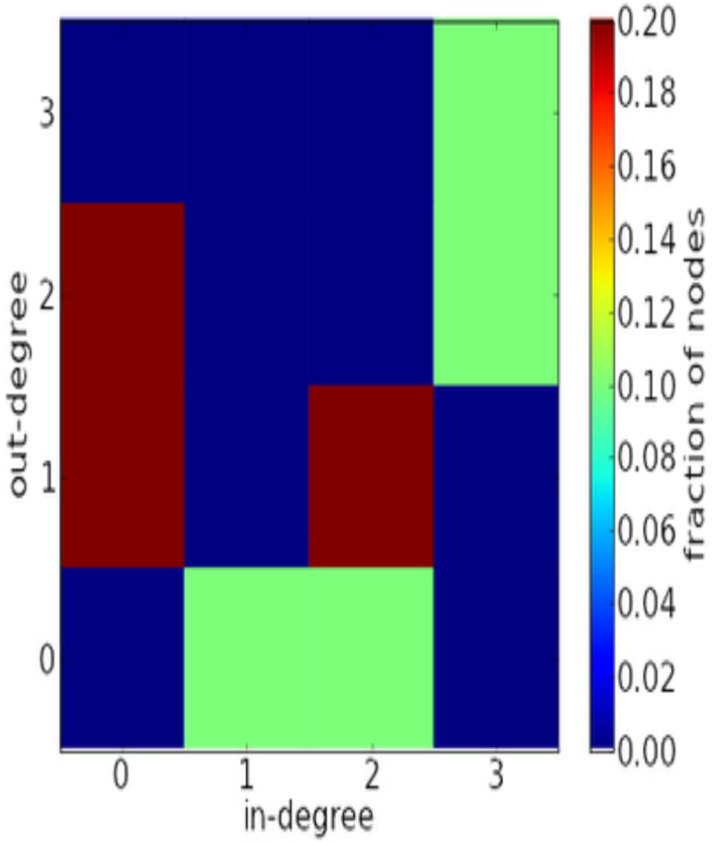
$$P_{\text{deg}}(k) = \text{fraction of nodes in the graph with degree } k.$$

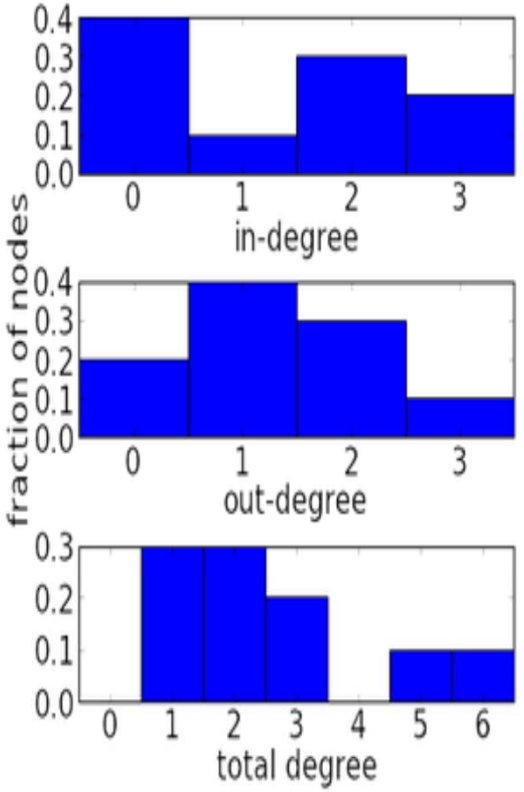


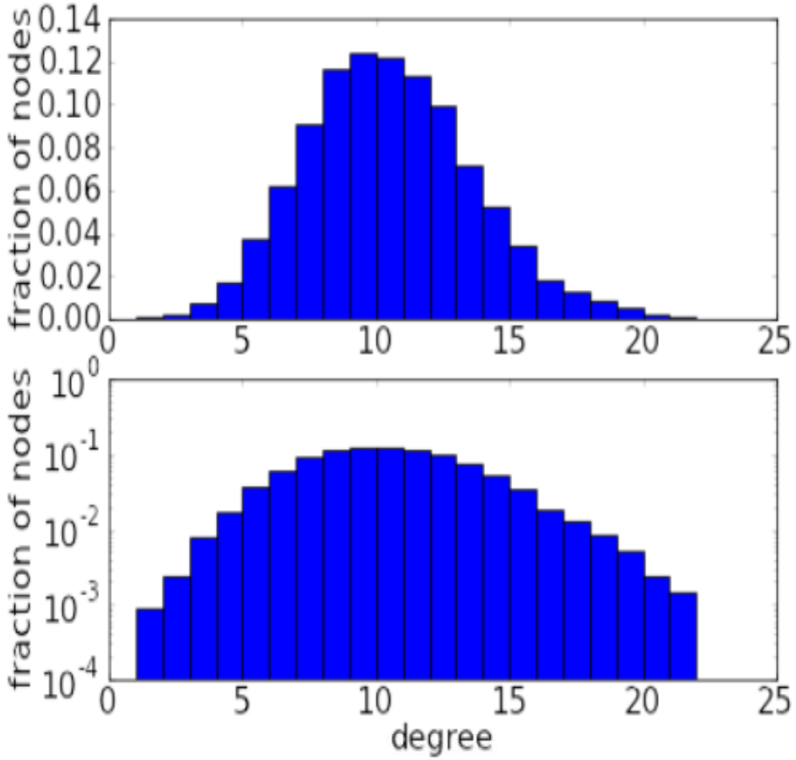


Degree Distribution

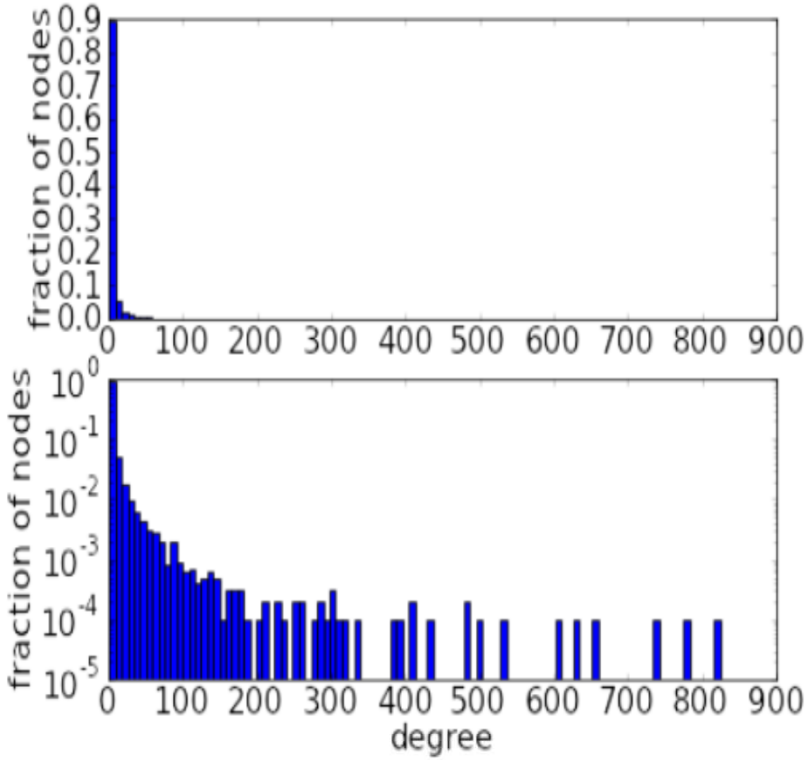








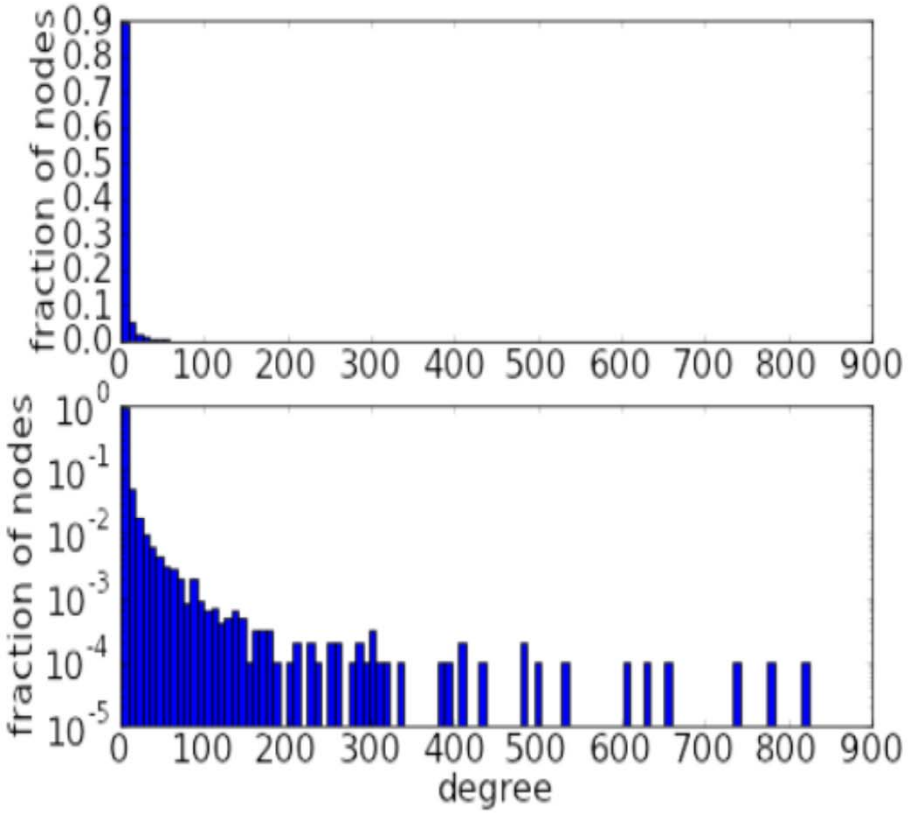
A binomial degree distribution of a network with 10,000 nodes and average degree of 10. The top histogram is on a linear scale while the bottom shows the same data on a log scale.



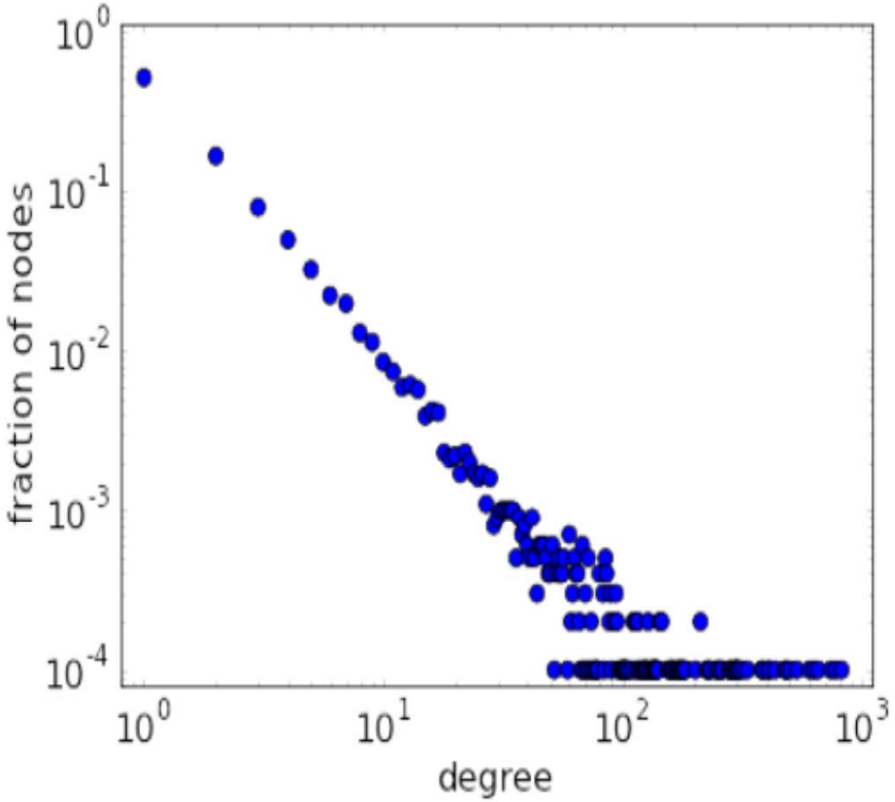
A power law degree distribution of a network with 10,000 nodes and average degree of around 7. The top histogram is on a linear scale while the bottom shows the same data on a log scale.

Scale-Free Network: The presence of nodes with a much higher degree than most other nodes

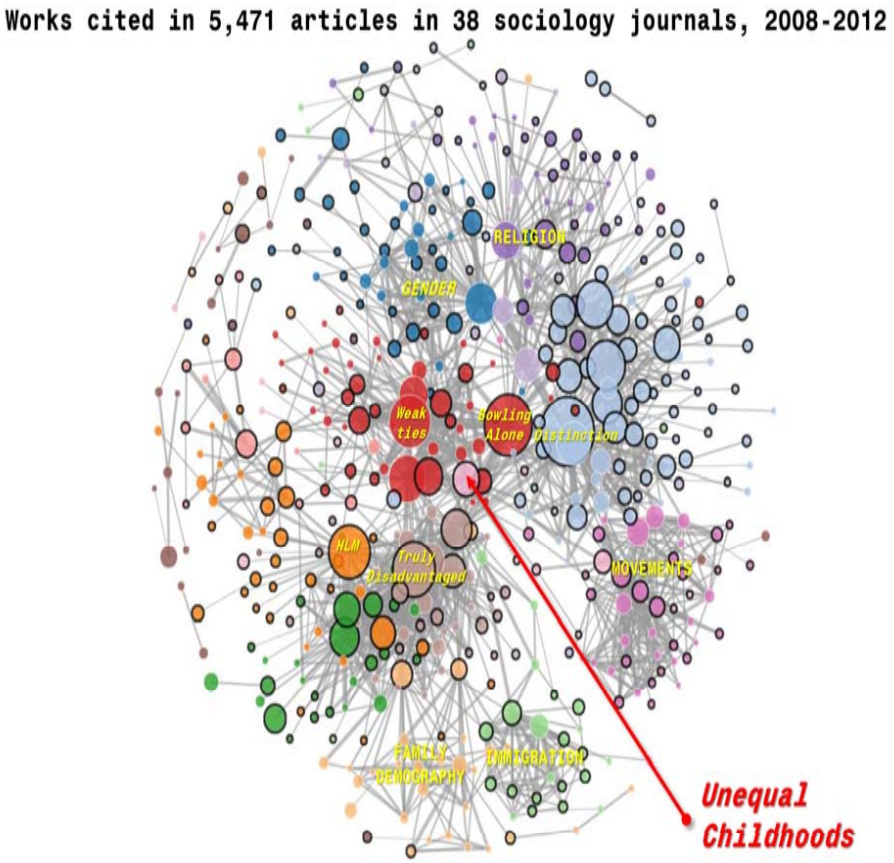
$$P_{\deg}(k) \propto k^{-\gamma}$$

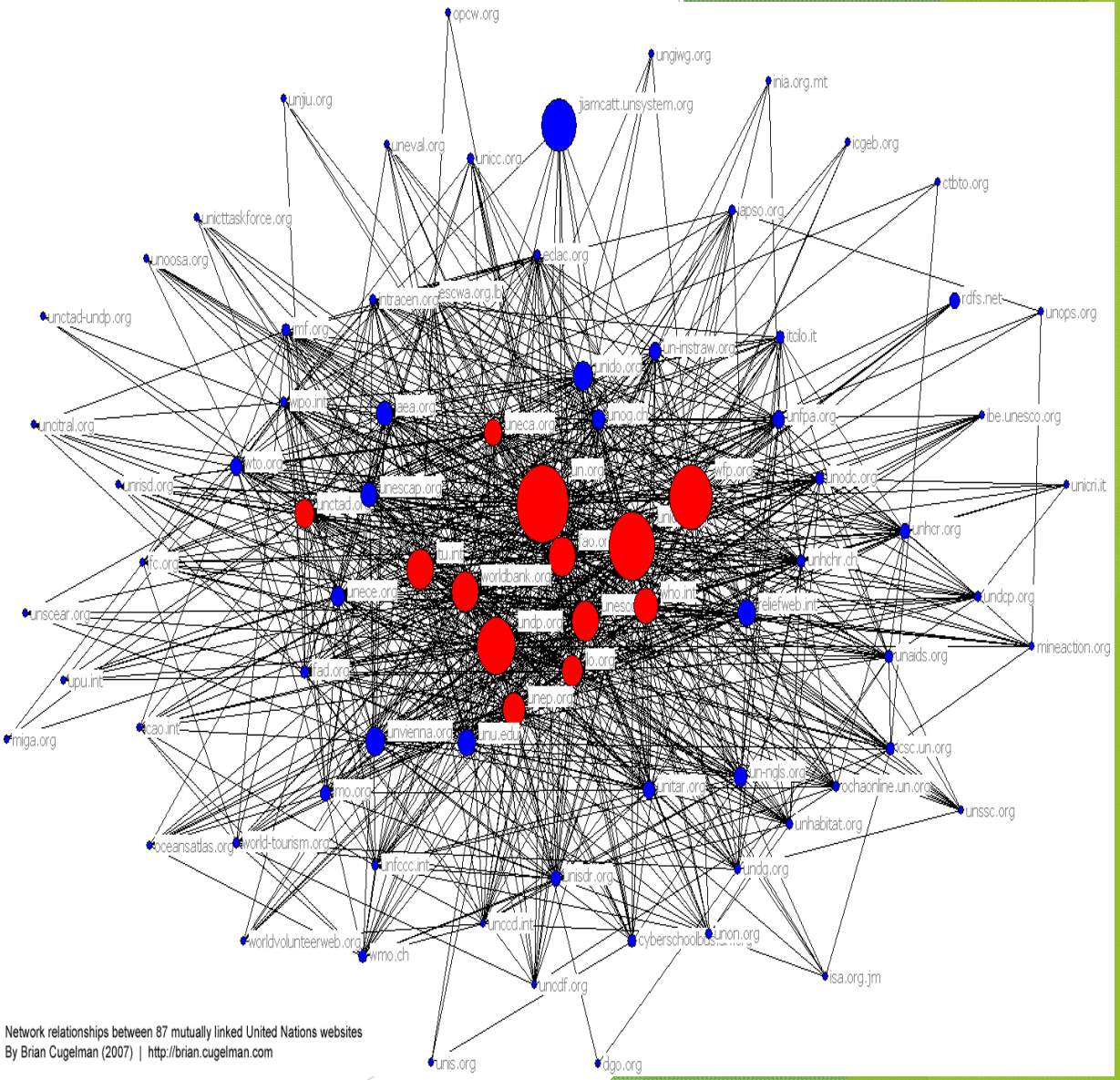


Log $P_{deg}(k) = - \gamma k + a$



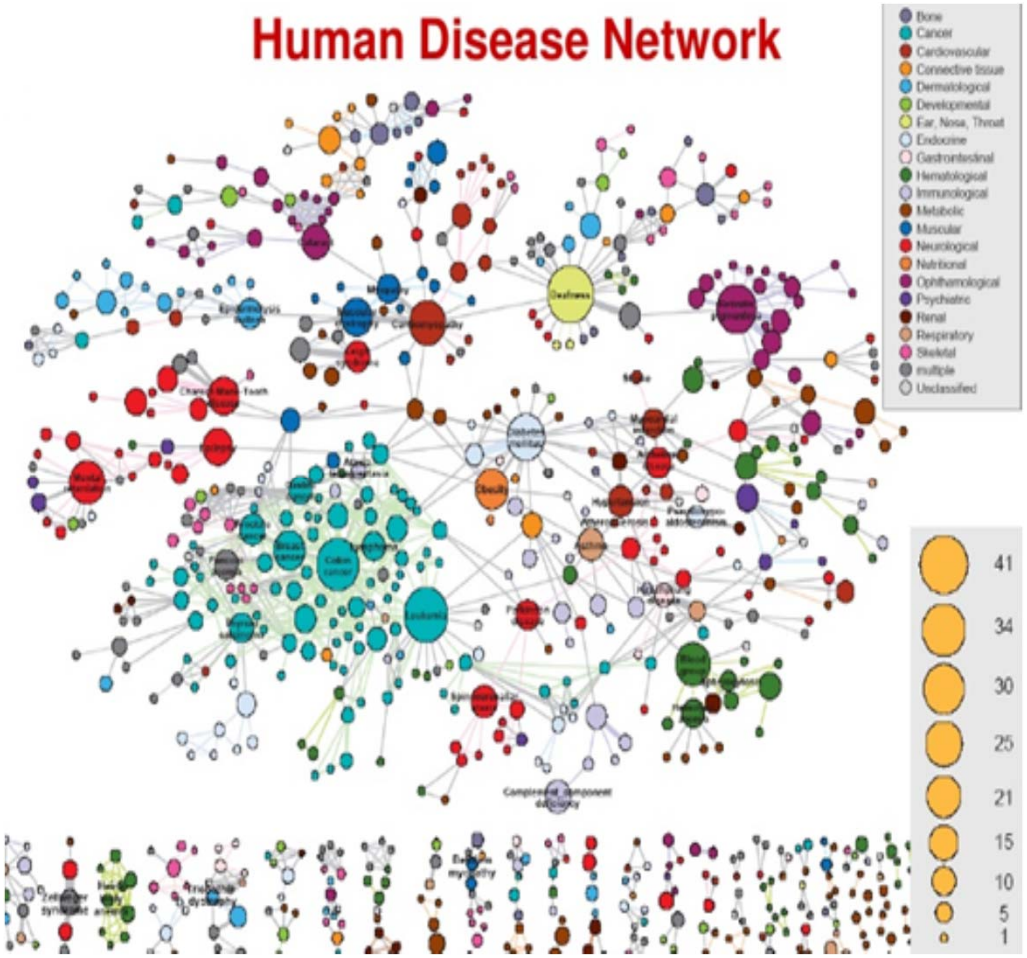
Centrality

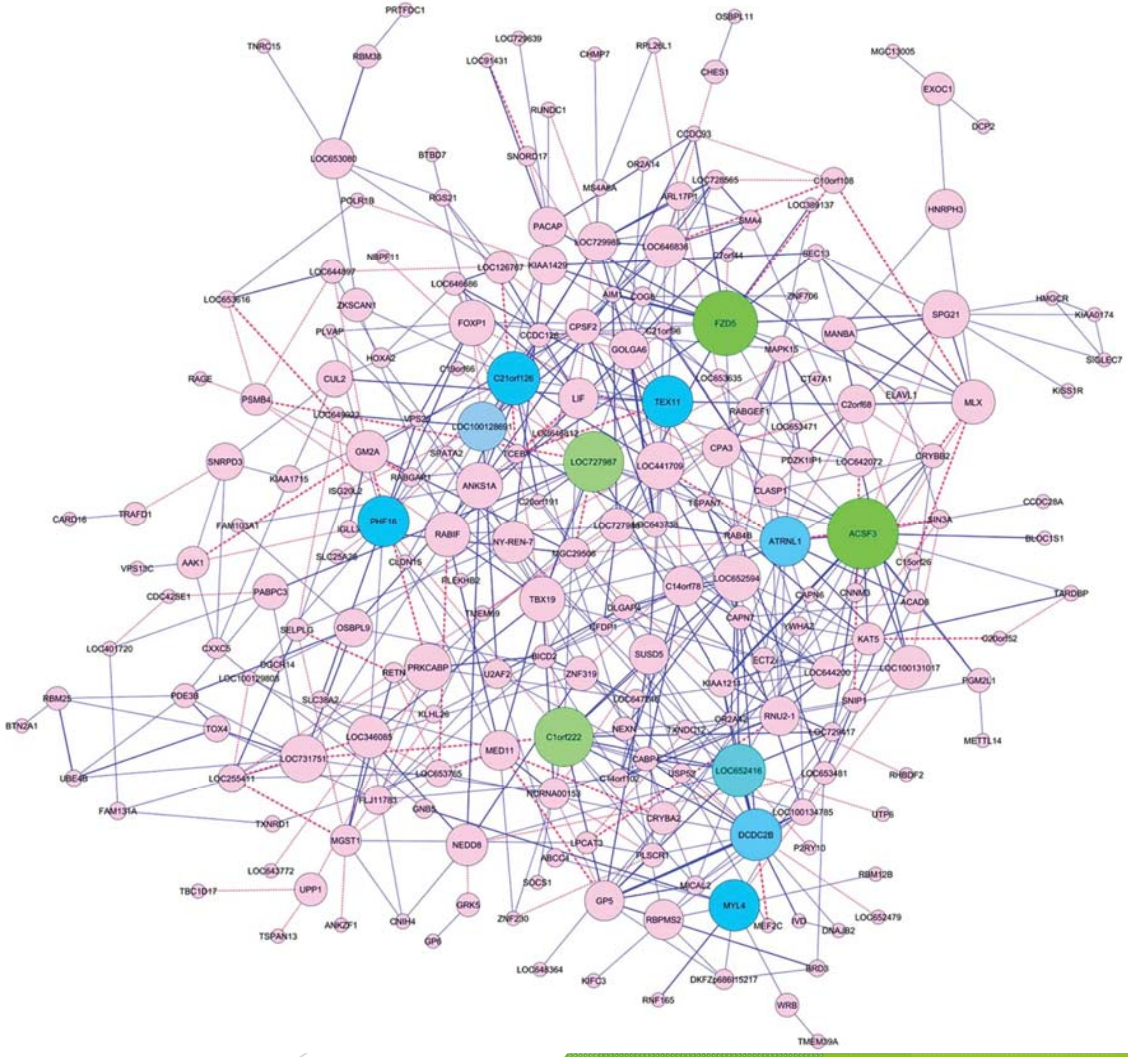




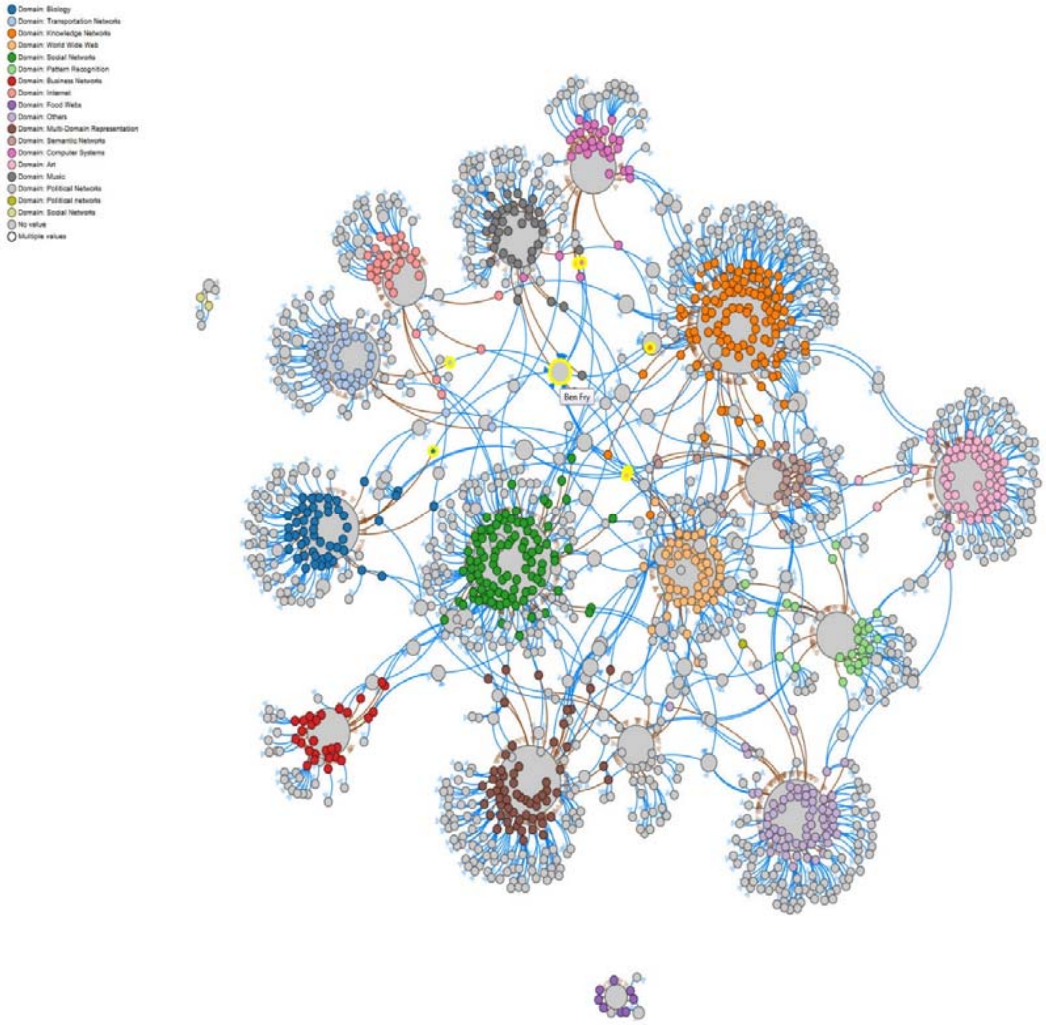
Scale free networks

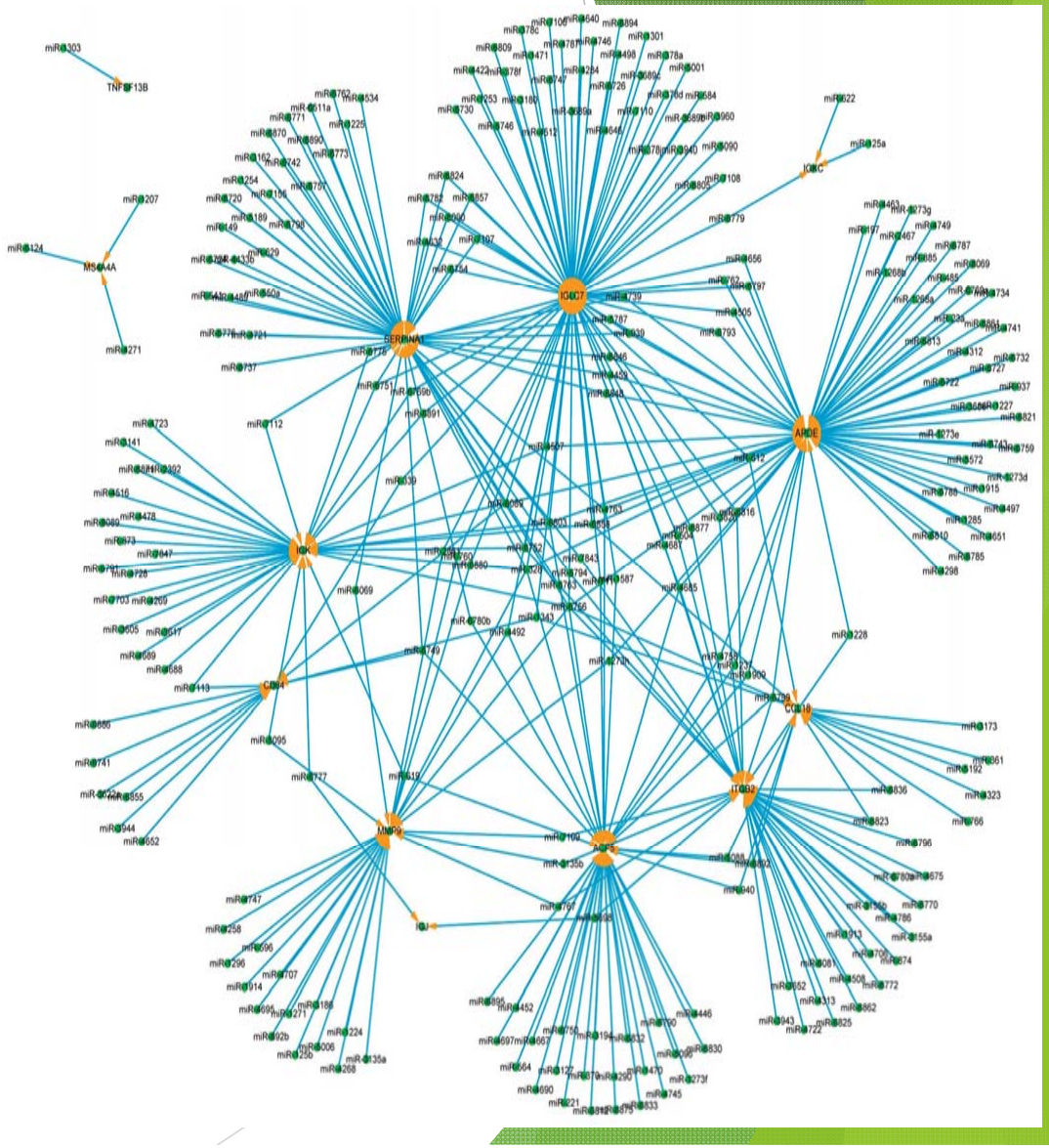
Centrality (Scale-free networks)



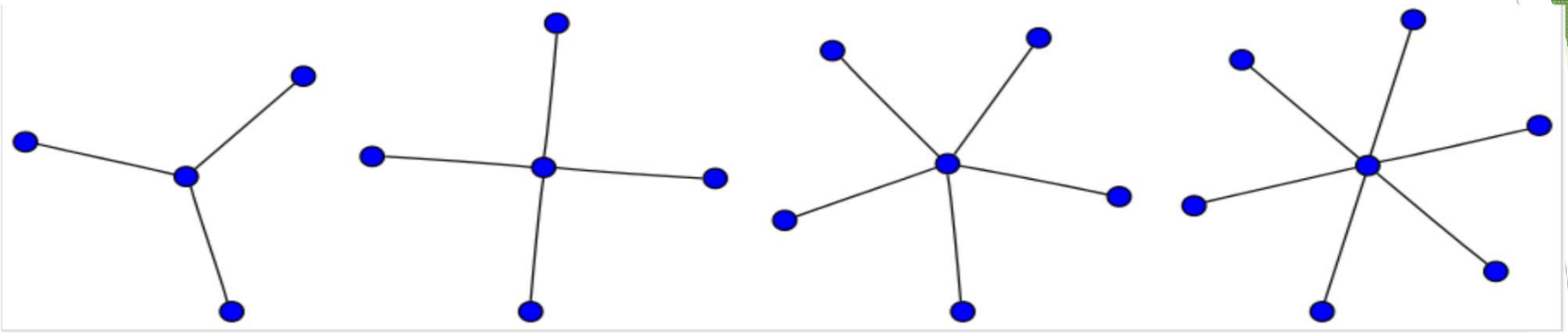


Visualization





Degree Centrality



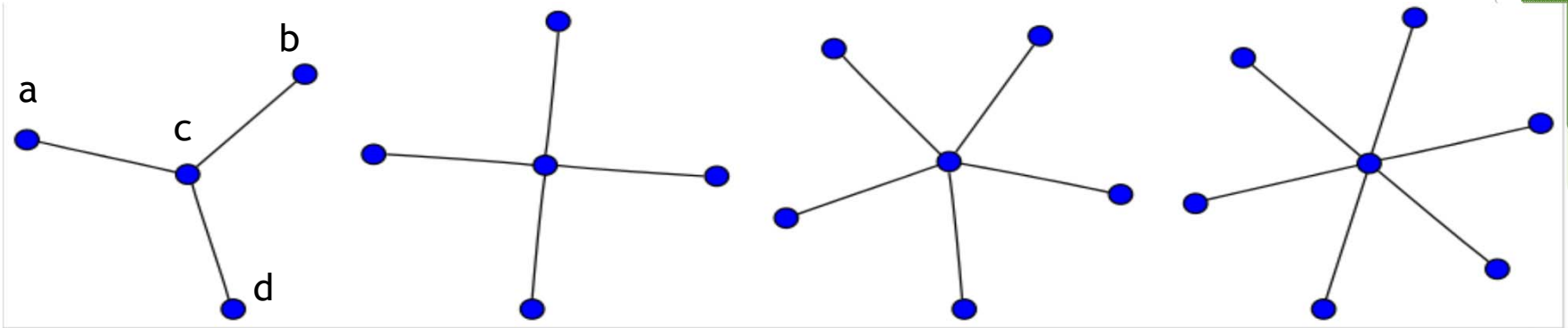
The star graphs S_3, S_4, S_5 and S_6.

Find a node x in a graph to maximize the score

$$\text{score}(x) = \sum_y \left| \text{degree}(x) - \text{degree (y is connected to x)} \right|$$

For star graphs S3, S4, S5, S6, their maximum values are 6,12,20,30, … ,(n-1)(n-2)

Degree Centrality



The star graphs S_3, S_4, S_5 and S_6.

Find a node x in a graph to maximize the score

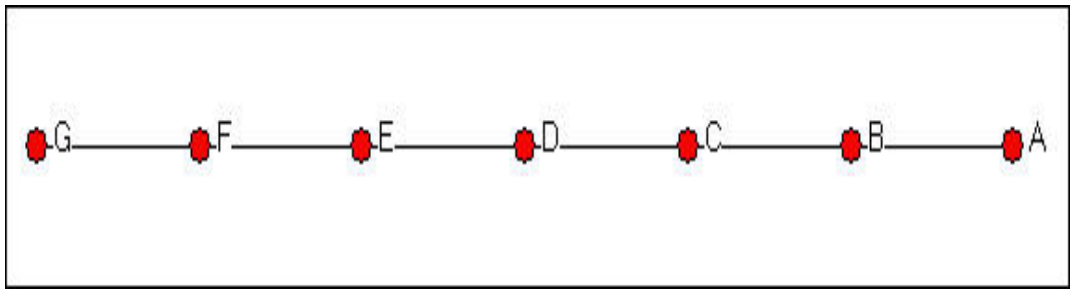
$$\text{score}(x) = \sum_y \left| \text{degree}(x) - \text{degree (y is connected to x)} \right|$$

degree(a)=degree(b)=degree(d)=1, degree(c)=3

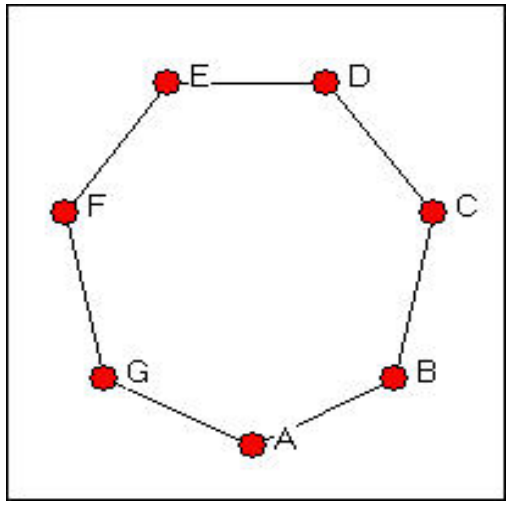
For star graph S3, take x=a, score(a)=|degree(a)-degree(c)|=2; take x=c, score(c)=|degree(c)-degree(a)|+|degree(c)-degree(b)|+|degree(c)-degree(d)|=6

Degree Centrality

- Line Network



- Circle Network



- What are their scores ?

Betweenness Centrality:

$$\text{Score}(i) = \sum_{j \neq k} g_{jk}(i) / g_{jk}(i)$$

$g_{jk}(i)$ = the number of shortest paths connecting jk passing through i

g_{jk} = total number of shortest paths

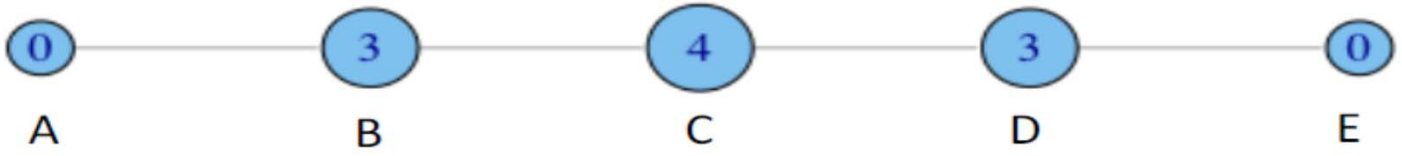
Example

A lies between no two other nodes

B lies between A and 3 other nodes: C,D and E

C lies between 4 pairs of vertices (A,D), (A,E), (B,D), (B,E)

Note that there are no alternative paths for these pairs to take, so C gets full credit



Closeness Centrality:

Closeness is based on the length of the average shortest path between a node and all nodes in the graph. The graph has N nodes, therefore, the average distance is equal to

$$\sum_{j=1}^{N} \text{distance}\,(A,j)/(N-1)$$

Closeness Centrality:

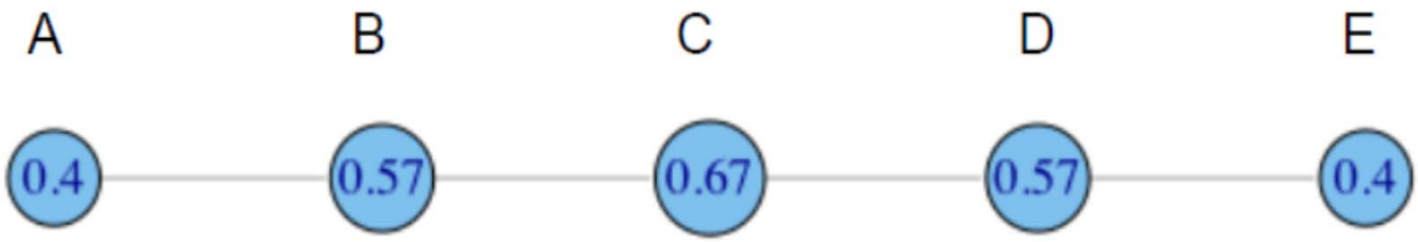
The average shortest path is equal to

$$\sum_{j=1}^{N} \text{distance}\,(A,j)/(N-1)$$

The reciprocal of this value is how close they are.

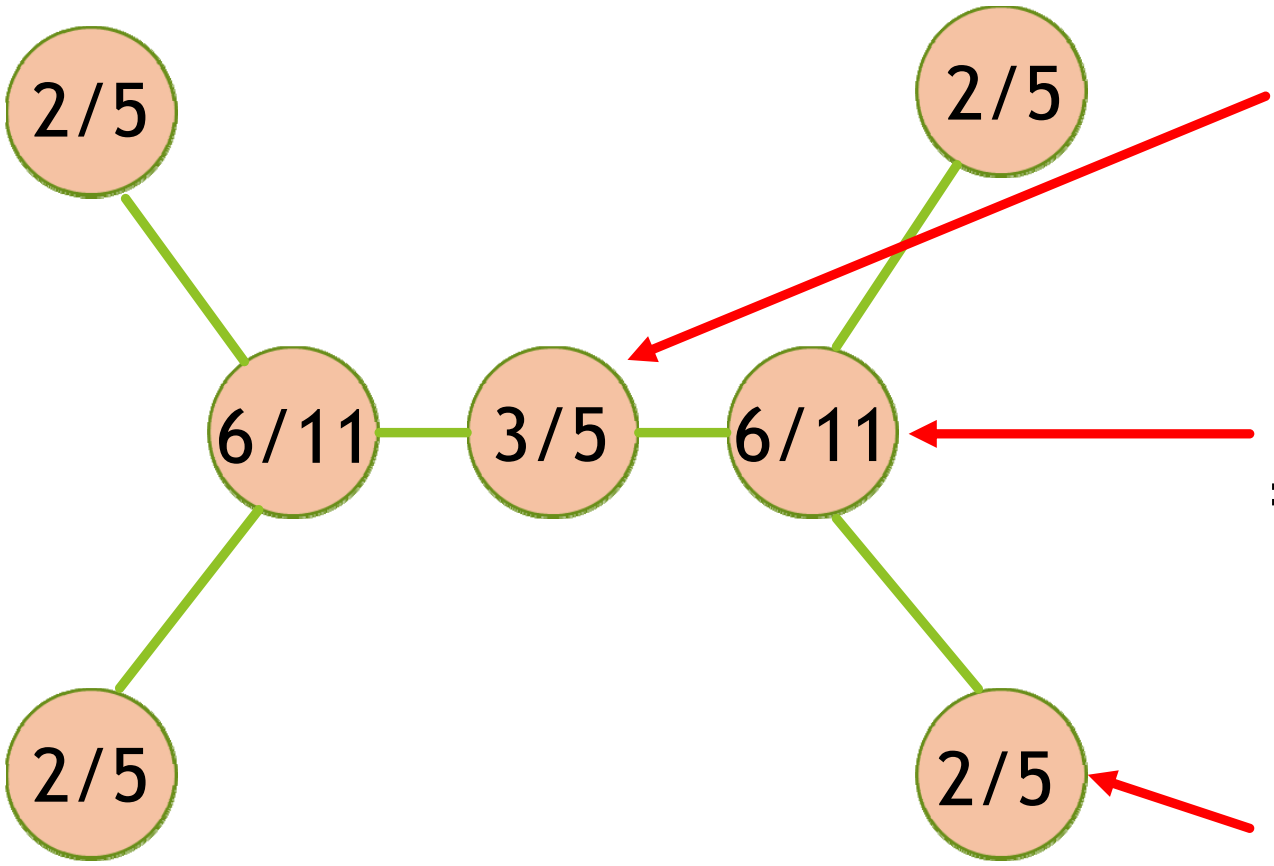
$$\left(\sum_{j=1}^{N} \text{distance}\,(A,j)/(N-1) \right)^{-1}$$

Example



$$\left[\frac{\sum_{j=1}^{N} d(A,j)}{N-1}\right]^{-1} = \left[\frac{1+2+3+4}{4}\right]^{-1} = \left[\frac{10}{4}\right]^{-1} = 0.4$$

Example



$((1+1+2+2+2+2)/6)^{-1}$
$=6/10=3/5$

$((1+1+1+2+3+3)/6)^{-1}$
$=6/11$

$((1+2+2+3+4+4)/6)^{-1}$
$=2/5$

Eigenvector Centrality

- Eigenvector centrality (also called eigencentrality) is a measure of the influence of a node in a network.
- There may be some nodes not high degree but they have big influence, see the previous example.
- It assigns relative scores to all nodes in the network based on the concept that connections to high-scoring nodes contribute more

  to the score of the node in question

  than equal connections to

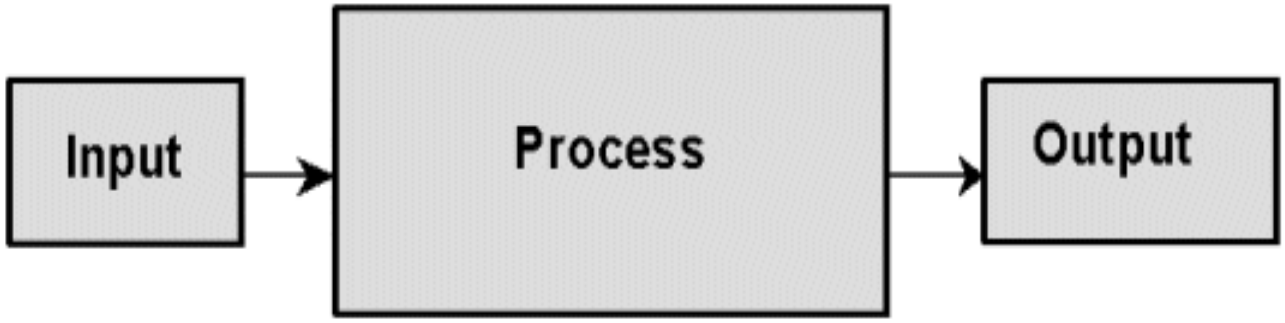
  low-scoring nodes.
- PageRank

  (Google Search Engine)

特徵值和特徵向量
(Eigenvalues and Eigenvectors)

Eigenvalues and Eigenvectors ? (特徵值和特徵向量 ?)

- Invariant (不變) in Input (輸入) and Output (輸出)



特徵值和特徵向量 (Eigenvalues and Eigenvectors)

- Invariant (不變) in Input (輸入) and Output (輸出)

$$x \longrightarrow M \longrightarrow M(x)=\lambda x$$

- M adjacency matrix (n rows and n columns); pairwise connections
- λ eigenvalue (a number)
- x eigenvector (a column of n numbers for n nodes)

特徵值和特徵向量 (Eigenvalues and Eigenvectors)

- What is the meaning of x ?
- What is the meaning of M(x) ?

特徵值和特徵向量 (Eigenvalues and Eigenvectors)

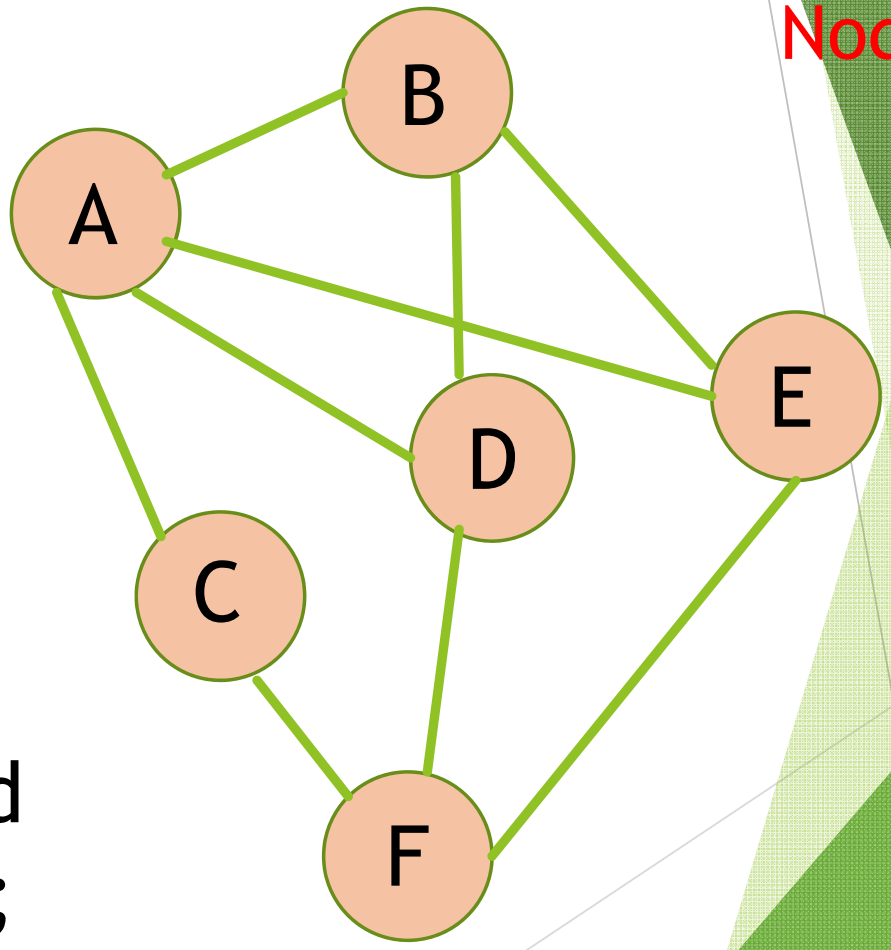
- What is the meaning of x ?

  Ans: nodal values

- What is the meaning of M(x) ? Ans: Re-distribution of nodal values

Re-distribution of nodal values

	A	B	C	D	E	F
A	---	1	1	1	1	0
B	1	---	0	1	1	0
C	1	0	---	0	0	1
D	1	1	0	---	0	1
E	1	1	0	0	---	1
F	0	0	1	1	1	---



Nodal values

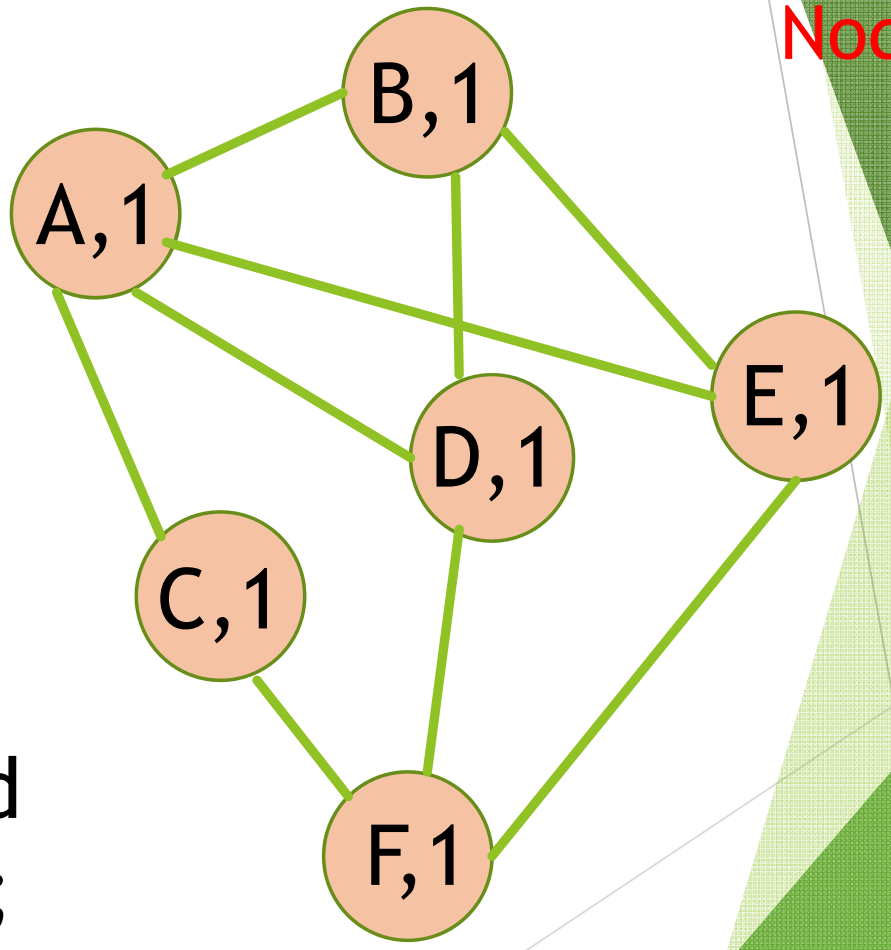
A	1
B	1
C	1
D	1
E	1
F	1

X

6 nodes; M: 6 rows and
6 columns; symmetric;
x is a column of 6 numbers.

Re-distribution of nodal values

	A	B	C	D	E	F
A	---	1	1	1	1	0
B	1	---	0	1	1	0
C	1	0	---	0	0	1
D	1	1	0	---	0	1
E	1	1	0	0	---	1
F	0	0	1	1	1	---



Nodal values

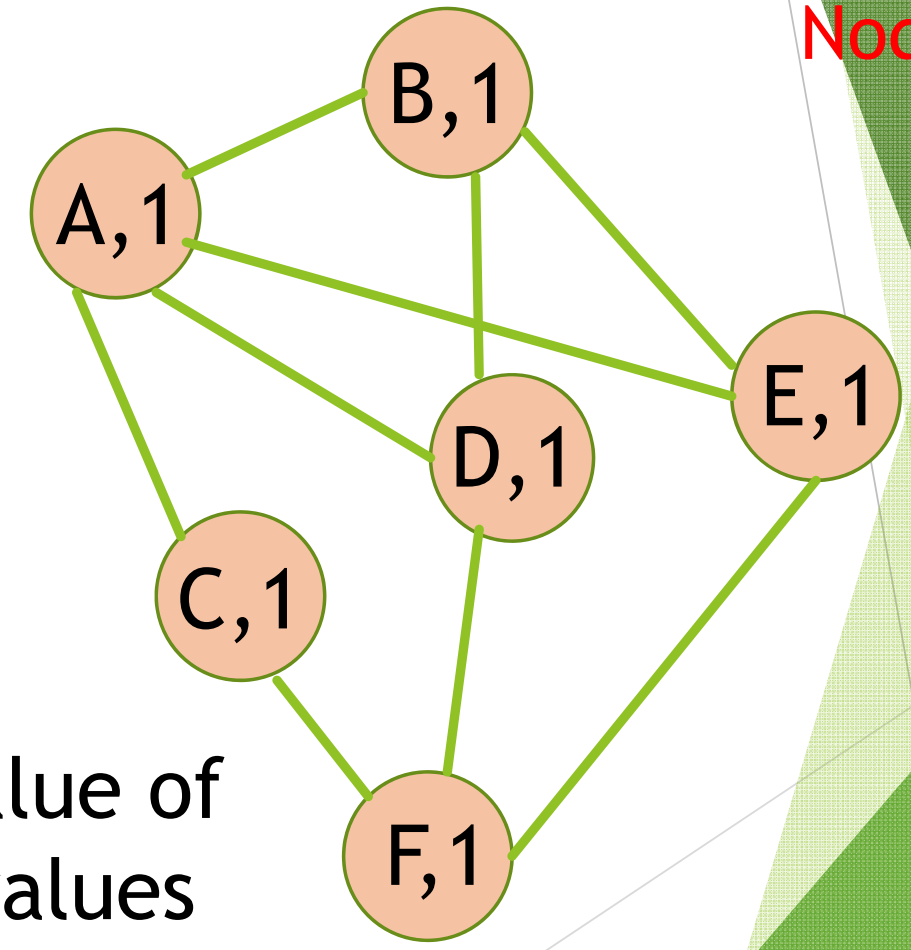
A	1
B	1
C	1
D	1
E	1
F	1

X

6 nodes; M: 6 rows and
6 columns; symmetric;
x is a column of 6 numbers

Re-distribution of nodal values

Node	connected from
A	B, C, D, E
B	A, D, E
C	A, F
D	A, B, F
E	A, B, F
F	C, D, E



Nodal values

A	1
B	1
C	1
D	1
E	1
F	1

X

Compute the nodal value of A by using the nodal values of B,C,D,E

Re-distribution of nodal values

Node connected from

A B, C, D, E

What are the contributions from B,C,D,E ?

B connects 3 nodes (A,D,E),

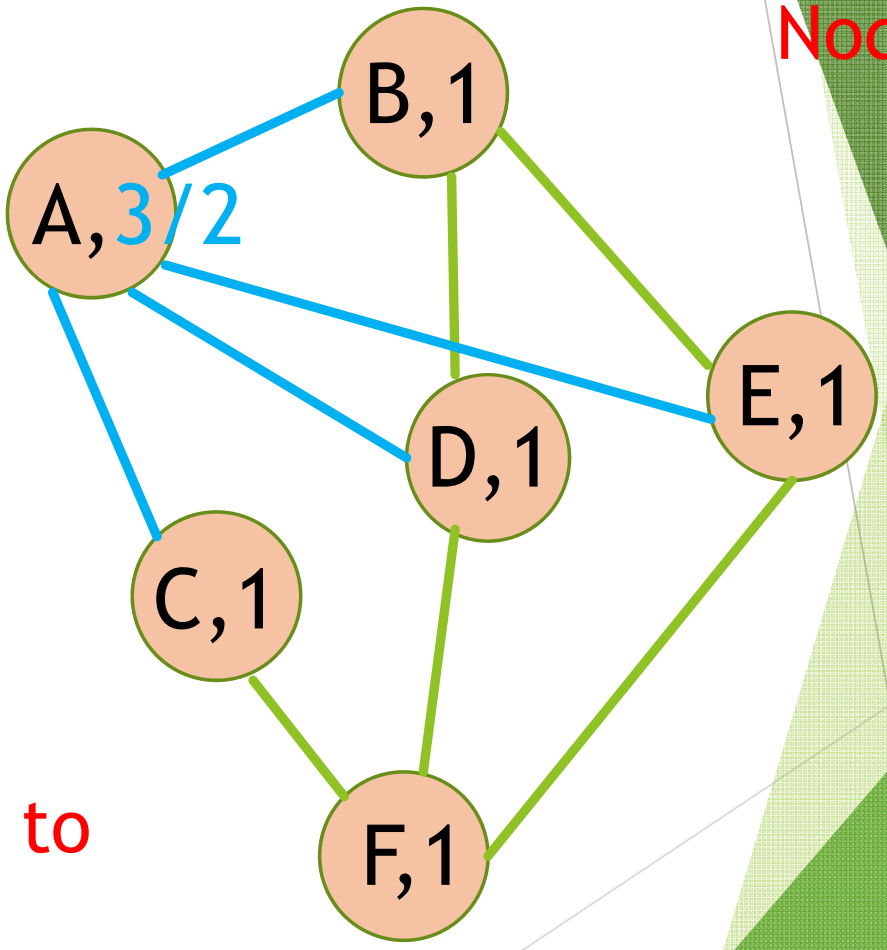
C connects 2 nodes (A,F),

D connects 3 nodes (A,B,F),

E connects 3 nodes (A,B,F).

The nodal value of A is equal to

1/3+1/2+1/3+1/3=3/2



Nodal values

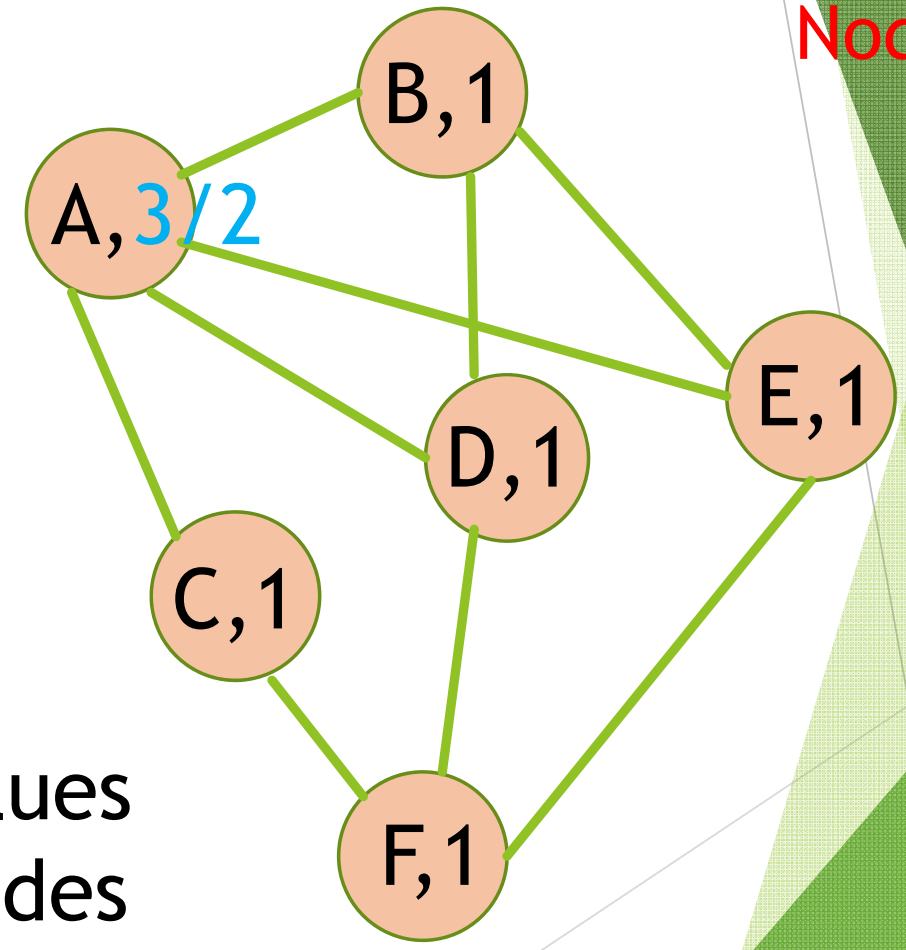
A	1
B	1
C	1
D	1
E	1
F	1

X

Re-distribution of nodal values

Node	connected from
A	B, C, D, E
B	A, D, E
C	A, F
D	A, B, F
E	A, B, F
F	C, D, E

Compute the nodal values via their connected nodes and their connections.



Nodal values

A	1
B	1
C	1
D	1
E	1
F	1

X

Re-distribution of nodal values

Node connected from

B A, D, E

What are the contributions from A,D,E ?

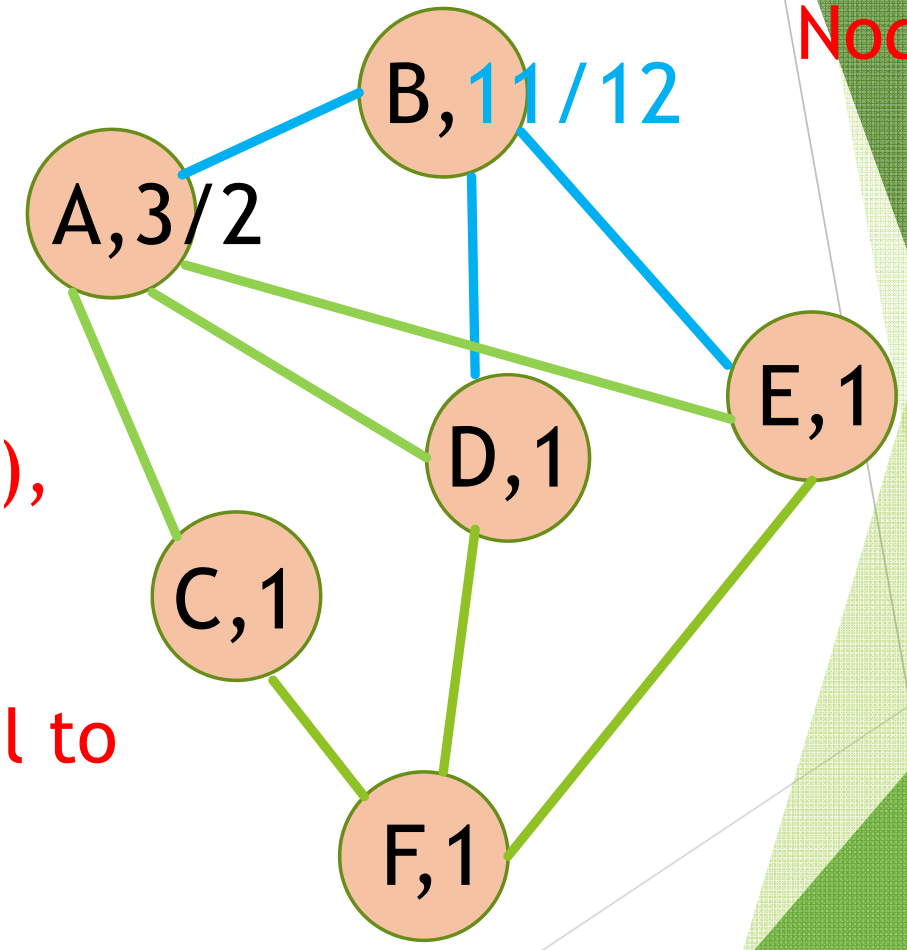
A connects 4 nodes (B,C,D,E),

D connects 3 nodes (A,B,F),

E connects 3 nodes (A,B,F).

The nodal value of B is equal to

1/4+1/3+1/3=11/12



Nodal values

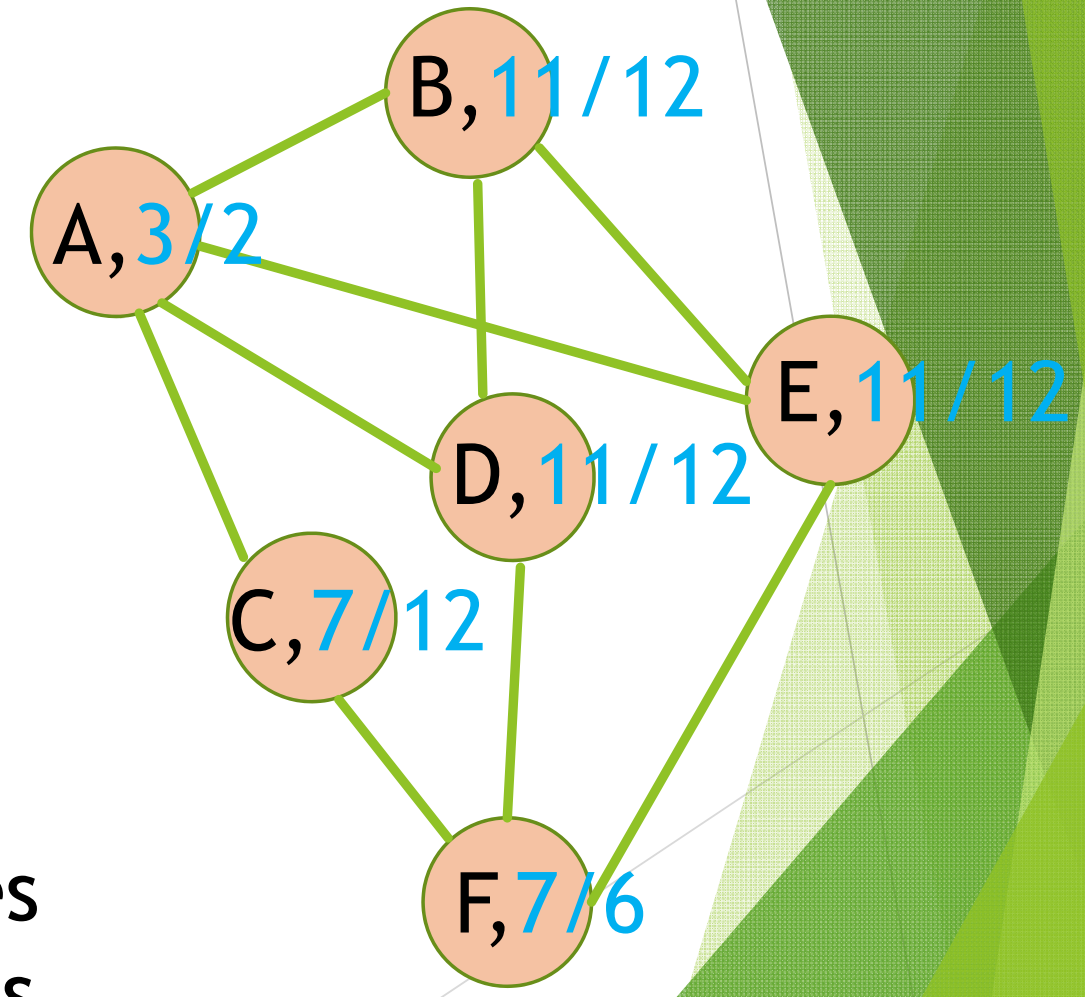
A	1
B	1
C	1
D	1
E	1
F	1

X

Re-distribution of nodal values

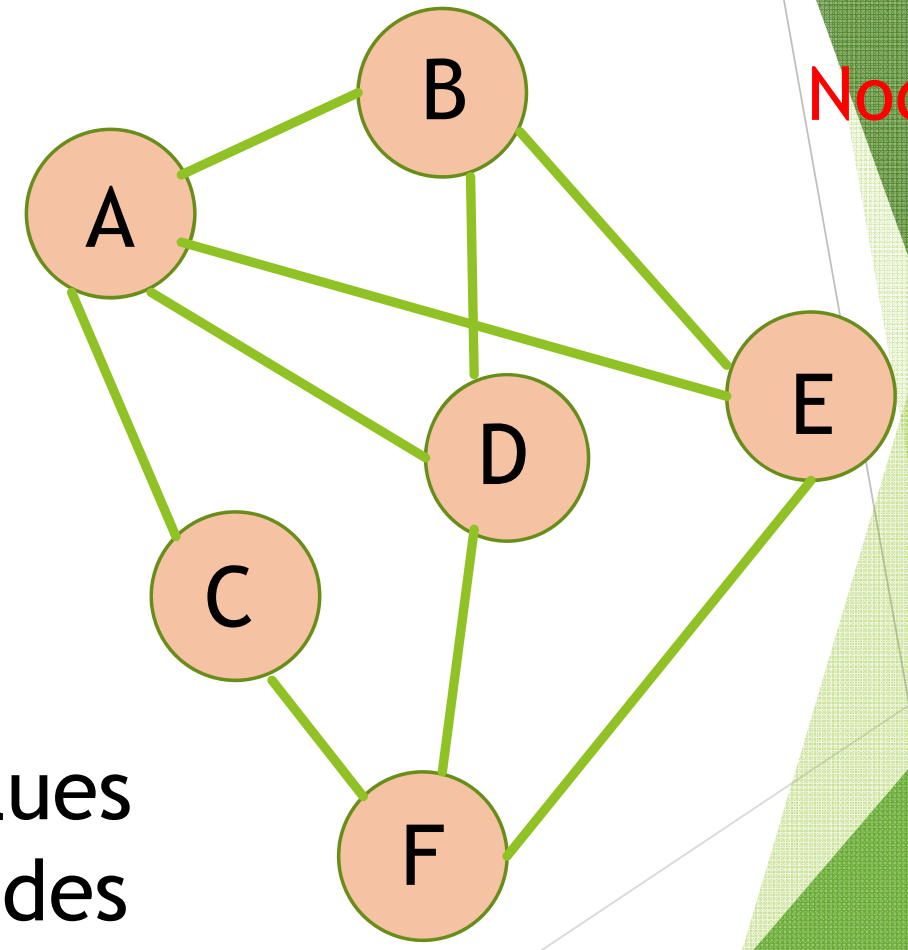
Node	Calculation
A	(1/3+1/2+1/3+1/3)=3/2
B	(1/4+1/3+1/3)=11/12
C	(1/4+1/3)=7/12
D	(1/4+1/3+1/3)=11/12
E	(1/4+1/3+1/3)=11/12
F	(1/2+1/3+1/3)=7/6

Compute the nodal values via their connected nodes



Re-distribution of nodal values

Node	connected from
A	B, C, D, E
B	A, D, E
C	A, F
D	A, B, F
E	A, B, F
F	C, D, E



Nodal values

A	3/2
B	11/12
C	7/12
D	11/12
E	11/12
F	7/6

x_{new}

Compute the nodal values via their connected nodes and their connections.

$M(x)=\lambda x$?

$M($

A	1
B	1
C	1
D	1
E	1
F	1

$)=$

A	3/2
B	11/12
C	7/12
D	11/12
E	11/12
F	7/6

is not equal to λ

A	1
B	1
C	1
D	1
E	1
F	1

It is not an eigenvector

Re-distribution of nodal values

Node connected from

A B, C, D, E

What are the contributions from B,C,D,E ?

B connects 3 nodes (A,D,E),

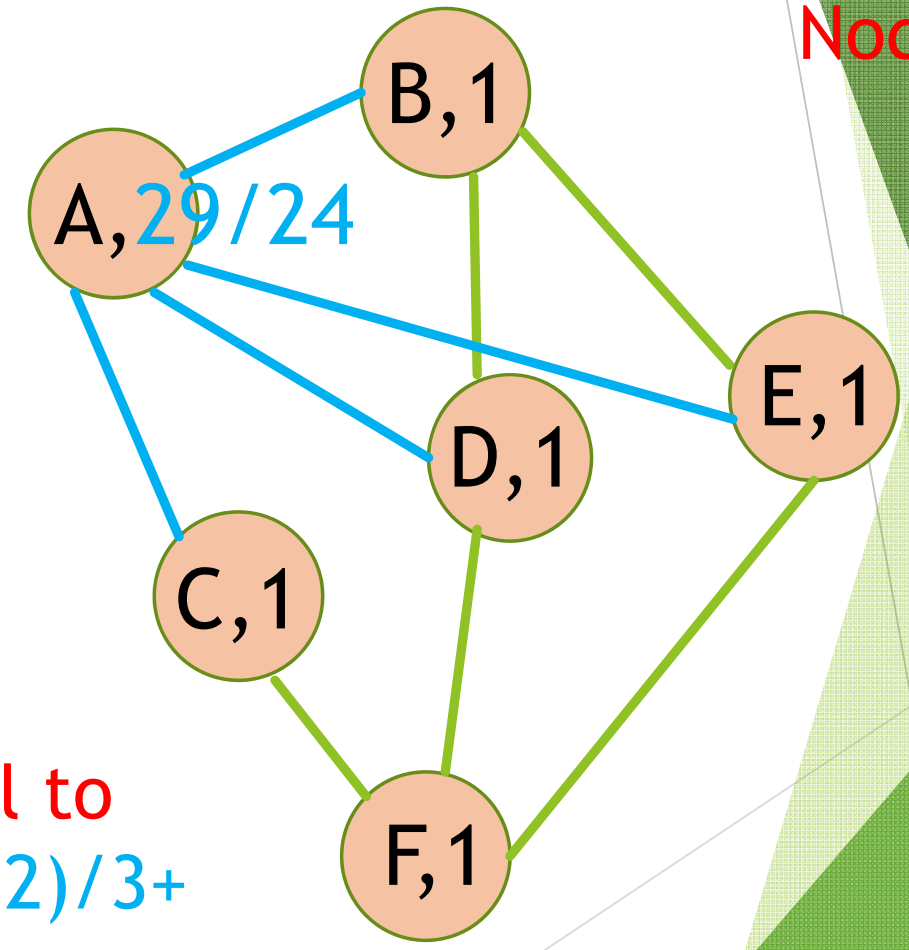
C connects 2 nodes (A,F),

D connects 3 nodes (A,B,F),

E connects 3 nodes (A,B,F).

The nodal value of A is equal to

(11/12)/3+(7/12)/2+(11/12)/3+(11/12)/3=29/24



Nodal values

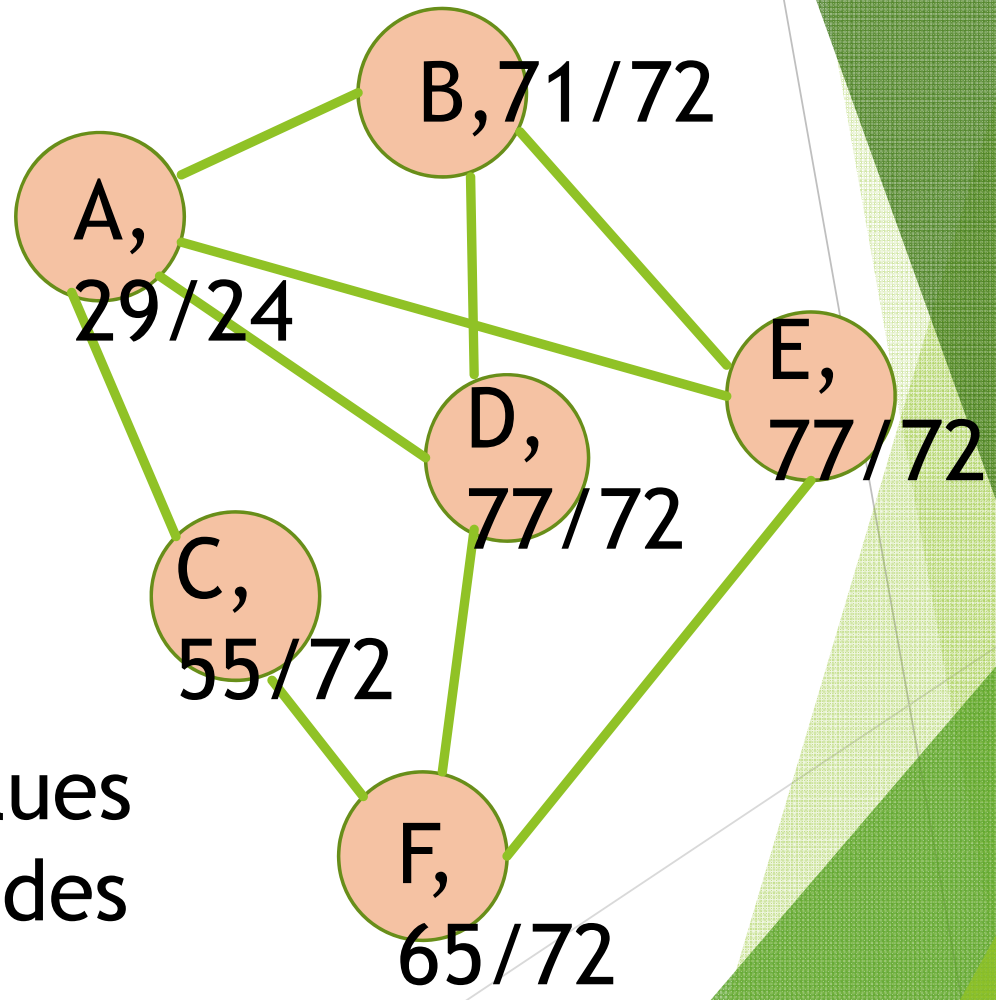
A	3/2
B	11/12
C	7/12
D	11/12
E	11/12
F	7/6

X

Re-distribution of nodal values

Node	connected from
A	B, C, D, E
B	A, D, E
C	A, F
D	A, B, F
E	A, B, F
F	C, D, E

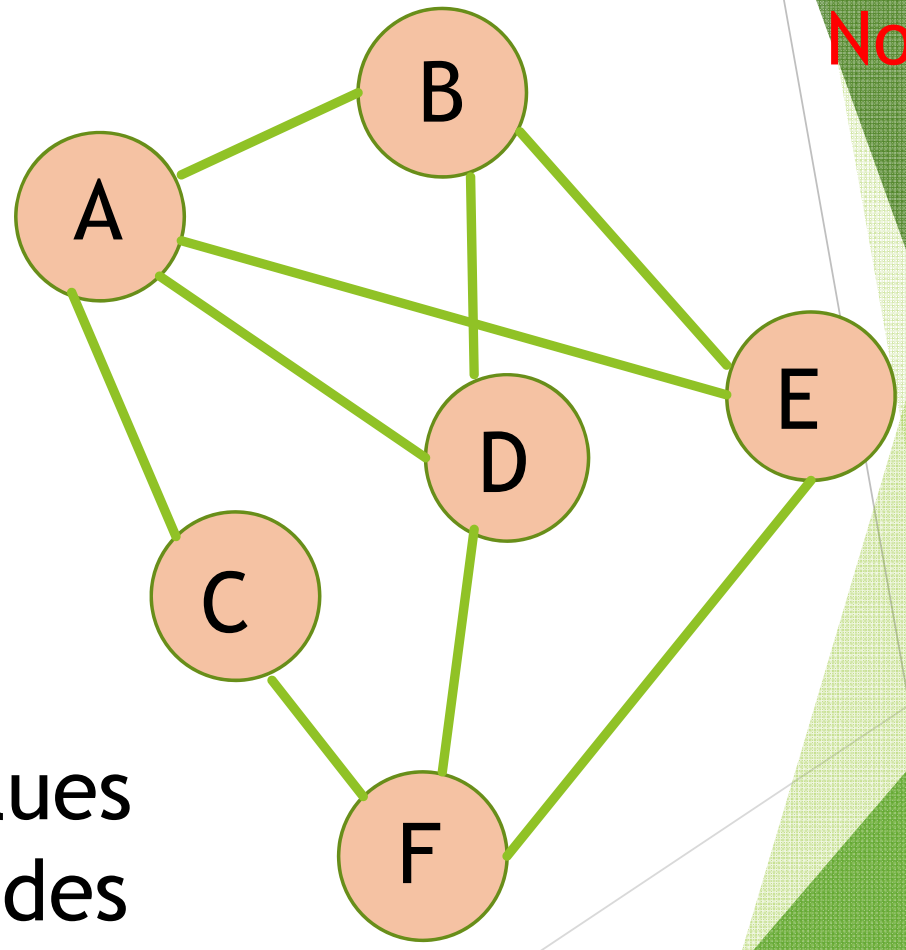
Compute the nodal values via their connected nodes and their connections.



Re-distribution of nodal values

Node	connected from
A	B, C, D, E
B	A, D, E
C	A, F
D	A, B, F
E	A, B, F
F	C, D, E

Compute the nodal values via their connected nodes and their connections.



Nodal values

A	29/24
B	71/72
C	55/72
D	77/72
E	77/72
F	65/72

X_{new}

$M(x)=\lambda x$?

$M($

A	3/2
B	11/12
C	7/12
D	11/12
E	11/12
F	7/6

$)=$

A	29/24
B	71/72
C	55/72
D	77/72
E	77/72
F	65/72

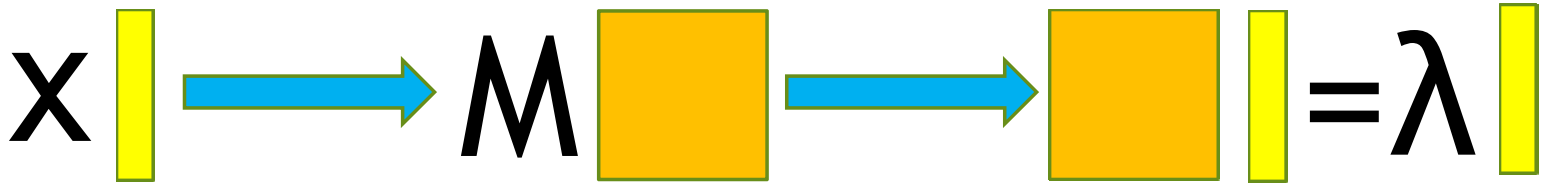
is not equal to λ

A	3/2
B	11/12
C	7/12
D	11/12
E	11/12
F	7/6

It is still not an eigenvector

特徵值和特徵向量 (Eigenvalues and Eigenvectors)

- Invariant in Input and Output



- M  adjacency matrix (n rows and n columns)
- x  eigenvector (a column of n numbers)
- λ eigenvalue (a number)

Perron-Frobenius Theorem
Oskar Perron (1907) and Georg Frobenius (1912)

M  all the numbers are nonnegative and all the nodes are connected together

- $\lambda = 1$
- For the corresponding column x  (the eigenvector), all the numbers are positive

Perron-Frobenius Theorem
Oskar Perron (1907) and Georg Frobenius (1912)

M  all the numbers are nonnegative,

and all the nodes are connected together

- x, M(x), M(M(x)), M(M(M(x))), M(M(M(x)))), … is very close to  the eigenvector

All the nodes are connected

B→A, C→A, D→A, E→A, F→C→A

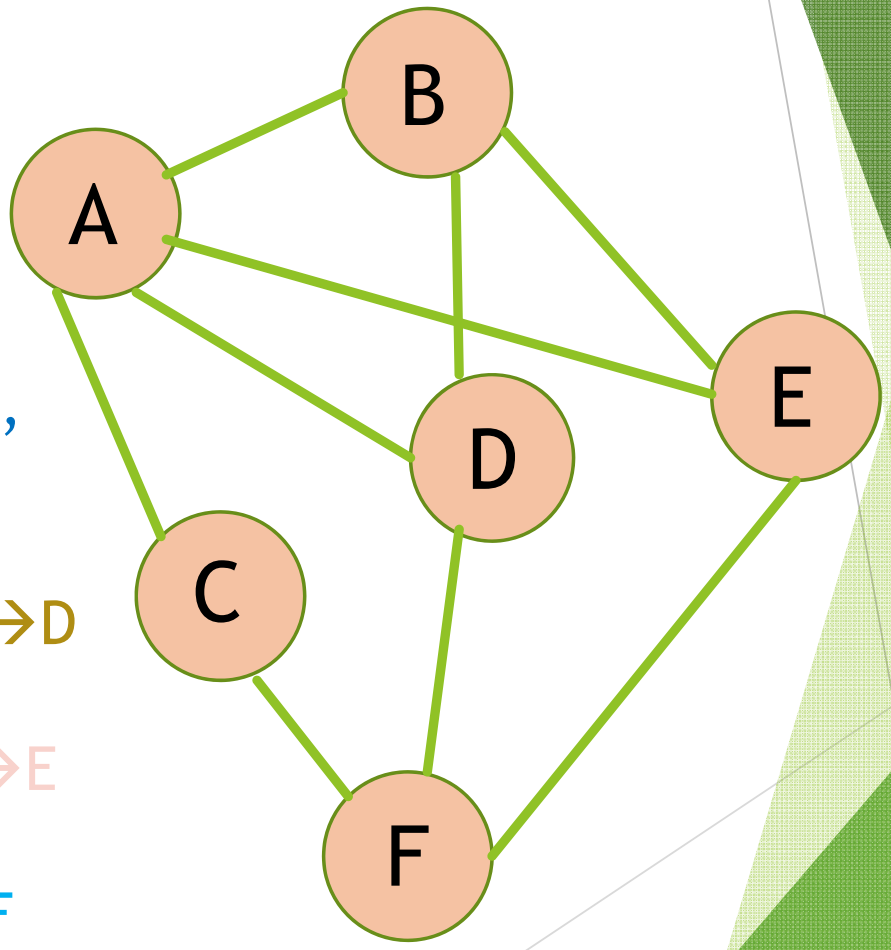
A→B, C→A→B, D→B, E→B,
F→D→B

A→C, B→A→C, D→A→C, E→A→C,
F→C

A→D, B→D, C→A→D, E→A→D, F→D

A→E, B→E, C→A→E, D→A→E, F→E

A→C→F, B→D→F, C→F, D→F, E→F



x, M(x), M(M(x)), …

$x=$

A	1
B	1
C	1
D	1
E	1
F	1

, $M(x)=$

A	3/2
B	11/12
C	7/12
D	11/12
E	11/12
F	7/6

, $M(M(x))=$

A	29/24
B	71/72
C	55/72
D	77/72
E	77/72
F	65/72

x, Mx, M(Mx), M(M(Mx)), …
Excel Calculation

Almost the same and positive

x	M(x)	M(M(x))	M(M(M(x)))	M(M(M(M(x))))	M(M(M(M(M(x)))))		M(…(M(x))…)	M(…(M(x))…)
1	1.5	1.2083333	1.42361111	1.26099537	1.39226466	…	1.3333334	1.3333334
1	0.916667	0.9861111	1.0150463	0.977044753	1.021395962	…	1.0000000	1.0000000
1	0.583333	0.7638889	0.60300926	0.720871914	0.622797711	…	0.6666667	0.6666667
1	0.916667	1.0694444	0.93171296	1.059220679	0.948479295	…	1.0000000	1.0000000
1	0.916667	1.0694444	0.93171296	1.059220679	0.948479295	…	1.0000000	1.0000000
1	1.166667	0.9027778	1.09490741	0.922646605	1.066583076	…	1.0000000	1.0000000

A	4/3
B	1
C	2/3
D	1
E	1
F	1

The number λ is 1 and Mx = 1x

M(x)=x YES !

M(

A	4/3
B	1
C	2/3
D	1
E	1
F	1

)= 1

A	4/3
B	1
C	2/3
D	1
E	1
F	1

			A 4/3 4 nodes	B 1 3 nodes	C 2/3 2 nodes	D 1 3 nodes	E 1 3 nodes	F 1 3 nodes
A	4/3	=		1 * 1/3 +	2/3 * 1/2 +	1 * 1/3 +	1 * 1/3	
B	1	=	4/3 * 1/4 +			1 * 1/3 +	1 * 1/3	
C	2/3	=	4/3 * 1/4 +					1 * 1/3
D	1	=	4/3 * 1/4 +	1 * 1/3 +				1 * 1/3
E	1	=	4/3 * 1/4 +	1 * 1/3 +				1 * 1/3
F	1	=			2/3 * 1/2 +	1 * 1/3 +	1 * 1/3	

Perron-Frobenius Theorem
Oskar Perron (1907) and Georg Frobenius (1912)

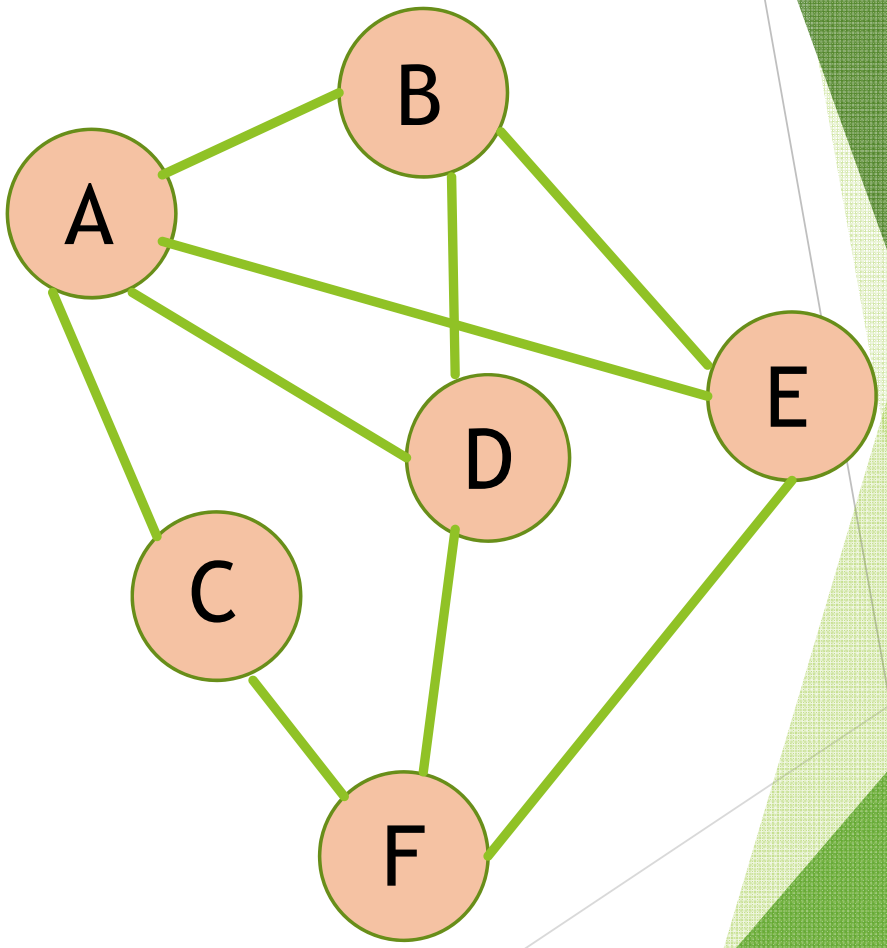
M  all the numbers are nonnegative,

and all the nodes are connected together

- x, M(x), M(M(x)), M(M(M(x))), M(M(M(x)))) … tend to the eigenvector  (as a limit 極限)

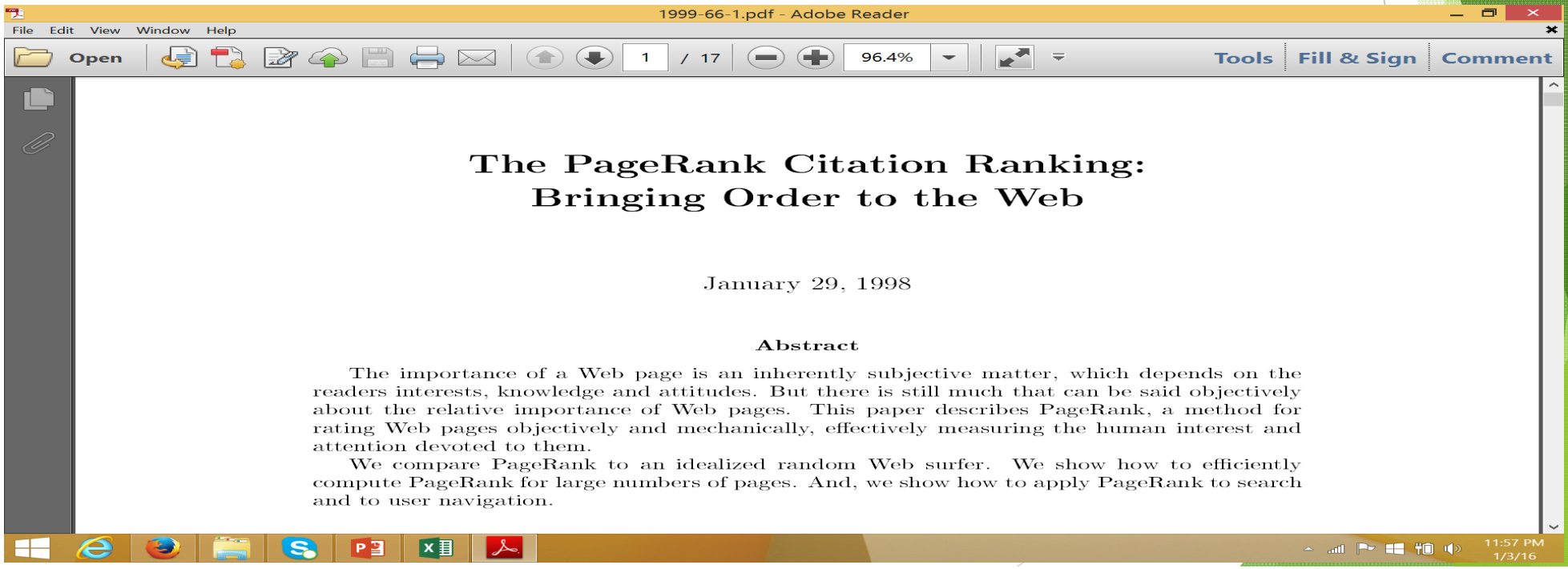
Eigenvector Centrality Ranking

Node	Eigenvector value	Ranking
A	4/3	1
B	1	2
C	2/3	3
D	1	2
E	1	2
F	1	2



谷歌搜索引擎

- Alphabet Inc. (Google) → Total Asset USD 148 billions
- Google Search Engine: Eigenvector Centrality Ranking
- Page, Lawrence; Brin, Sergey; Motwani, Rajeev and Winograd, Terry



谷歌搜索引擎

- Nodes ←→ Webpages
- Edges ←→ Hyperlinks
- Crawling Technology: identifies hyperlinks and build a BIG network

Member. He is also the Engineering Panel Member of the Hong Kong Research Grants Council in the GRF Panel (2009-2015) and in Joint Research Schemes Panel (2016-). His research Google citations and ISI indices can be found in Google Scholar, Thomson Reuters, and Scopus.

At 2014, Michael obtained Hong Kong Baptist University President's Awards for Outstanding Performance in Scholarly Work and here is the related HKBU news . Michael won the Honourable Mention of Householder Award IX , in 1996 at Switzerland, an excellent young researcher's presentation at Nanjing International Conference on Optimization and Numerical Algebra, 1999, and the Outstanding Young Researcher Award of the University of Hong Kong , 2001. Michael and his collaborators (Tony Hu, F. Wu and B. Sokhansan) won the outstanding research paper on Artificial Intelligence "Mining and Dynamic Simulation of Sub-Networks from Large Biomolecular Networks" in the 2007 World Congress in Computer Science, Computer Engineering and Applied Computing. He is the first one in the Mathematics Departments of Hong Kong to be the Principal Investigator of an Innovation and Technology Funding Project (Tier 2) at 2003, and to be the Director of Croucher Foundation Advanced Study Institute at 2008. In 2010 and 2011, he obtains support from the Innovation and Technology Funding to work with Ma Belle Jewellery Company for a research project Sales Knowledge Acquisition and Customer Behavior Analysis Platform For Jewelry Retail Industry (about 1.3M) and to work a research project Heterogeneous Information Fusion Toolkit for Customer Behavior Acquisition and Modeling in Middle to High End Retail Industry (about 1M). Related information has been used in the video of the Faculty of Science. He also receives a US patent for a new and useful invention: Method and Device for Use in Converting a Colour Image into a Grayscale Image (US 8,355,566). His research in image stitching has been posted in SIAM Nuggets (October 2013): Seamless photography: using mathematical models for image stitching.

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He is an IEEE Senior Member.
He is also the Engineering Panel Member of the Hong Kong Research Grant
Council in the GRF Panel (2009-2015) and in
Joint Research Schemes Panel (2016-).
His research Google citations and ISI indices can be found in
<a href="http://scholar.google.com.hk/citations?user=BBpjLiIAAAAJ&hl=en">
Google Scholar, </a>
<a href="http://www.researcherid.com/rid/B-7189-2009">
Thomson Reuters, </a> and
<a href="http://www.scopus.com/authid/detail.url?authorId=34571761900">
Scopus. </a>

<p>
At 2014, Michael obtained
<a href="http://paward.hkbu.edu.hk/2014/scholarlywork1.html">
Hong Kong Baptist University
President's Awards for Outstanding Performance
in Scholarly Work </a>
and here is
<a href="http://hkbuenews.hkbu.edu.hk/?t=enews_details/526">
the related HKBU news </a>.
Michael won <a href="http://www.math.hkbu.edu.hk/~mng/house.gif"> the Honourable Mention of Householder
Award IX </a>, in 1996 at Switzerland,
an excellent young researcher's
presentation at Nanjing International Conference on Optimization and Numerical
Algebra, 1999, and
<a href="http://www.math.hkbu.edu.hk/~mng/mngout.jpg">
the Outstanding Young Researcher Award of the University of Hong Kong </a>,
2001.
Michael and his collaborators (Tony Hu, F. Wu and B. Sokhansan)
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Perron-Frobenius Theorem
Oskar Perron (1907) and Georg Frobenius (1912)

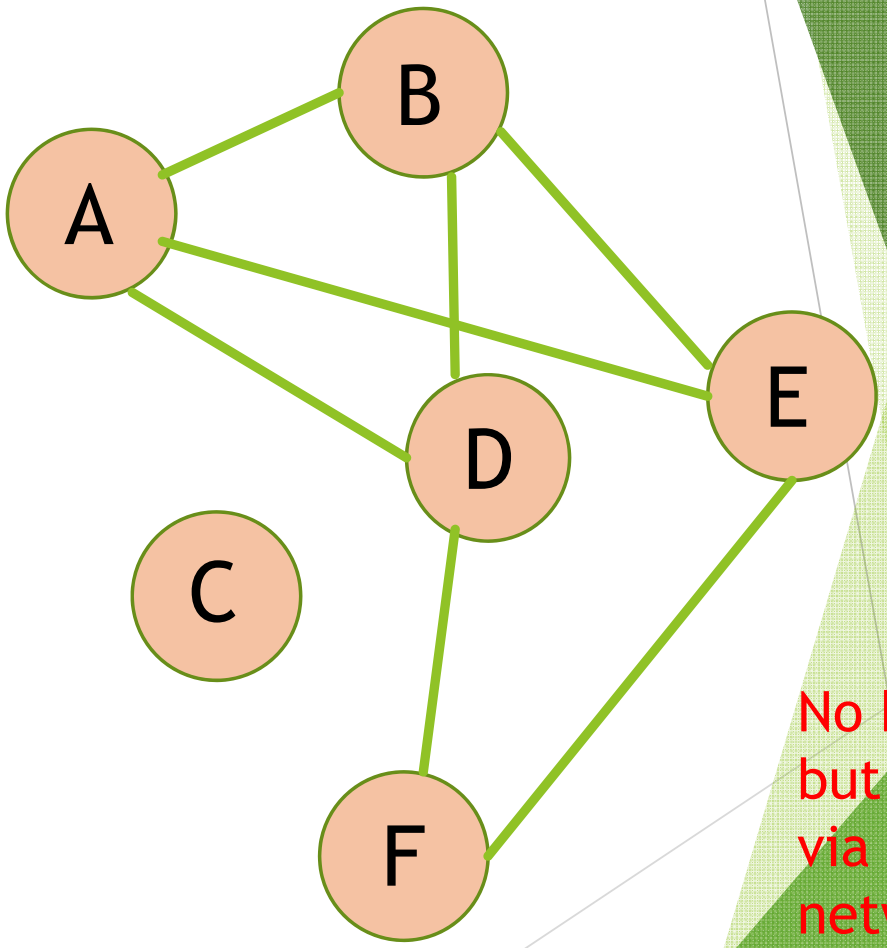
M  all the numbers are nonnegative,

and all the nodes are connected together

- x, M(x), M(M(x)), M(M(M(x))), M(M(M(x)))) … tend to the eigenvector  (as a limit)

How to guarantee all the nodes (webpages) are connected together in ?

	A	B	C	D	E	F
A	---	1	0	1	1	0
B	1	---	0	1	1	0
C	0	0	---	0	0	0
D	1	1	0	---	0	1
E	1	1	0	0	---	1
F	0	0	0	1	1	---



No hyperlink, but still reach via physical network

PageRank

- Technology: Crawling (hyperlink) + Implementation of WWW matrix
- Knowledge: Mathematics, graph theory and the theorem
- Innovation (創新,創意,創業)

(complete graph) a number

	A	B	C	D	E	F
A	---	1	0	1	1	0
B	1	---	0	1	1	0
C	0	0	---	0	0	0
D	1	1	0	---	0	1
E	1	1	0	0	---	1
F	0	0	0	1	1	---

+ α

	A	B	C	D	E	F
A	1	1	1	1	1	1
B	1	1	1	1	1	1
C	1	1	1	1	1	1
D	1	1	1	1	1	1
E	1	1	1	1	1	1
F	1	1	1	1	1	1

Perron-Frobenius Theorem
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M  all the numbers are nonnegative,

and all the nodes are connected together

- x, M(x), M(M(x)), M(M(M(x))), M(M(M(x)))) … tend to the eigenvector  (as a limit)

My Experience: ShowMuse at HK

Mobile adaptive learning
Recommendation system
Ranking engine

鼓勵應用數據科技　為中小企升級轉型
浸大數學系成功與業界代表 學習平台 ShowMuse™
產學研合作


新經濟新天地

「泛教育」自學App 提升社交技巧

ShowMuse課程種類

人際關係 Family and Relationship

自我增值 Self Improvement

創業 Entrepreneurship

飲食知識 F&B Knowledge

線上學習 方便上班族「進修」

「糖不甩」為青年開性教育課

A23 am NEWS

本地 由香港科技園投入5,000萬元起動的「科技企業投資基金」(Corporate Venture Fund, CVF)，自去年5月推出後，終完成首個投資項目。該家獲基金投資的公司為教學應用程式的初創企業，獲注資金額達300萬元。

教學App獲注資300萬

科技企業投資基金首投項目

CVF以配對形式運作，旨在吸引更多投資，向正在參與科技園培育計劃，已畢業及園區內的企業，在A輪或更早期的投資，提供進一步的資金支援。科技園創業培育計劃主管莫偉軒(圓圓)昨日表示，每個個案必須經由科技園及獨立專家組成的投資委員會審查，需時約6至9個月，包括深入調查財政狀況及管治架構等。

他續稱，基金已和約50家初創企業接觸，分別來自資訊科技、綠色科技、生物醫藥、有助發展智慧城市的物料與精密工程等領域，當中10至15家仍在商議中，預計有2至3家將交予投資委員會跟進。至於ShowMuse因何成為首家獲CVF注資，莫偉軒指其商業模式可本地化，有潛力打入內地市場，並且有先發優勢。

前新聞之花胞弟 患血癌萌創業意念

ShowMuse創辦人張龍華為前新聞之花張寶華胞弟，他坦言創業過程中獲姊姊協助，主要提供由她擔任導師的教學影片及介紹人脈等，惟資金籌集則全由他獨力承擔，初創時與拍檔合資6位數字金額，後來獲機構投資者投資300萬元，再獲CVF配對等額投資。公司創辦僅4個月，暫未收支平衡，期望2017年中達標。

他表示，萌生手機學習平台的念頭，始於10年前患血癌期間，醫生提醒他別把自己當病人，為了分散注意力，便想到找興趣課程學習。他笑說：「非學術課程，看似是雕蟲小技，但不學就很『擲使』。」他猶記得過去跟上司見客時，試過不懂唸紅酒名稱，便想到學品酒，怎料課程收費昂貴，令他察覺學習資源不平均問題。後來他的病治愈，仍對於該想法念念不忘，至近年手機及網絡發展成熟，遂於去年決心創業。

他分析其應用程式的優勢指，雖然現於美國、台灣及內地都有非學術課程的手機學習平台，卻沒有像他們般在課程影片內彈出選擇題測試學員，以及網內學員即時通訊程式。此外，學員完成課程及測試合格後會獲授虛擬勳章，達到要求才可參加線下活動，如品酒或咖啡班等。

ShowMuse創辦人張龍華稱，創業過程艱辛，因為曾經患病才鍛練出較強的心理承受能力。(受訪者提供)

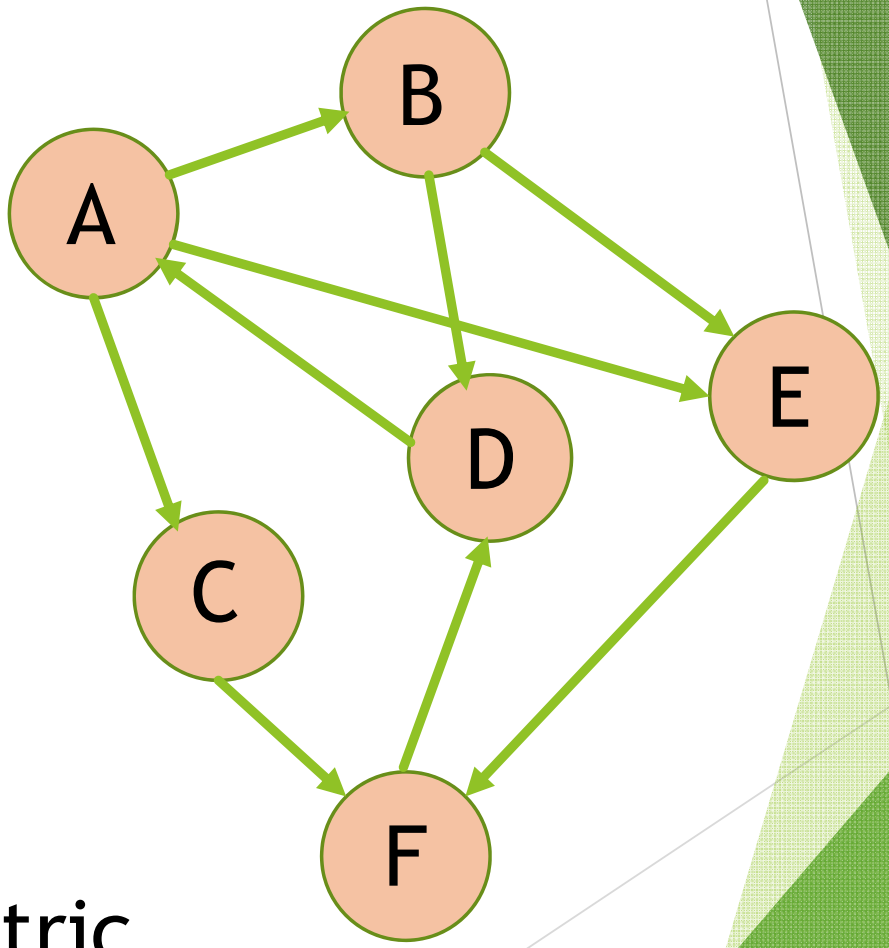
My Sharing

- "The bringing together of theory and practice leads to the most favourable results; not only does practice benefit, but the sciences themselves develop under the influence of practice, which reveals new subjects of investigation and new aspects of familiar subjects." P.L. Chebyshev

- Mathematics (graph theory) → PageRank (Applications) → new mathematics and computer algorithms (new research in tensors, high-order PageRank) → …

Exercise: Eigenvalue and Eigenvector ?

	A	B	C	D	E	F
A	---	0	0	1	0	0
B	1	---	0	0	0	0
C	1	0	---	0	0	0
D	0	1	0	---	0	1
E	1	1	0	0	---	0
F	0	0	1	0	1	---



6 nodes; 6 rows and
6 columns; not symmetric

Thank you very much !

Q/A