

Book of Abstracts

**International Workshop *on*
Numerical Analysis and Computational
Methods *for* Functional Differential and
Integral Equations**

3-6 December, 2007
Hong Kong Baptist University

Organizers:

Hermann BRUNNER
Department of Mathematics
Hong Kong Baptist University
Kowloon Tong
Hong Kong SAR
China
hbrunner@math.hkbu.edu.hk
hermann@math.mun.ca

Tao TANG
Department of Mathematics
Hong Kong Baptist University
Kowloon Tong
Hong Kong SAR
China
ttang@hkbu.edu.hk

Leevan LING
Department of Mathematics
Hong Kong Baptist University
Kowloon Tong
Hong Kong SAR
China
lling@hkbu.edu.hk

Chengjian ZHANG
Department of Mathematics
Huazhong University of Science and
Technology
Wuhan 430074
China
cjzhang@mail.hust.edu.cn

Sponsors:

Department of Mathematics, Hong Kong Baptist University
K.C. Wong Education Foundation
Lee Hysan Foundation
PKU-HKBU Joint Research Institute for Applied Mathematics, Hong Kong Baptist University

Contact Information:

Department of Mathematics
Hong Kong Baptist University
Kowloon Tong
Hong Kong SAR, China
Website: www.math.hkbu.edu.hk
Phone: (852) 3411-2348
Fax: (852) 3411-5811
Email: cchui@hkbu.edu.hk (Claudia Chui)



香港浸會大學
HONG KONG BAPTIST UNIVERSITY



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Conference Programme

Time	December 3	December 4	December 5	December 6
08:45	<i>Opening ceremony</i>			
09:00	Shoufu LI	S. VANDEWALLE	Tao TANG	Hongjjiong TIAN
	Chengming HUANG	Chengjian ZHANG	Emiko ISHIWATA	Yoshiaki MUROYA
	(Break)	(Break)	(Break)	
	Toshiyuki KOTO	Ming-Zhu LIU	H. BRUNNER	<i>Closing Ceremony</i>
12:00	(Lunch)	(Lunch)	(Lunch)	(Lunch)
14:00	Xiaomei QU	Sui Sun CHENG	(Discussion)	
	F. TEODORO	Yanping CHEN		
	(Break)	(Break)		
	M. Stela REBELO	Wansheng WANG		
16:30		Xiang XU	<i>Tour</i>	
18:00		<i>Banquet</i>		



Conference Programme — December 3 (Mon) —

08 : 45 – 09 : 00 **LT1, Cha Chi-ming Science Tower**

.....
OPENING CEREMONY
.....

09 : 00 – 09 : 50 **LT1, Cha Chi-ming Science Tower**

Shoufu LI

Contractivity and asymptotic stability properties of Runge-Kutta methods for Volterra functional differential equations

09 : 55 – 10 : 45

Chengming HUANG

Delay-dependent stability of numerical methods for delay differential equations

.....
Morning break
.....

11 : 10 – 12 : 00 **LT1, Cha Chi-ming Science Tower**

Toshiyuki KOTO

Stability of IMEX methods for ordinary and delay differential equations

12 : 00 – 14 : 00 **Staff Dining Room, Sir Run Run Shaw Building**

.....
Lunch
.....

14 : 00 – 14 : 30 **LT1, Cha Chi-ming Science Tower**

Xiaomei QU

Application of PPMLR scheme to the numerical simulation of fluid instability in inertial confinement fusion

14 : 35 – 15 : 05

Filomena TEODORO

A new approach to the numerical solution of forward-backward equations

.....
Afternoon break
.....

15 : 35 – 16 : 05 **LT1, Cha Chi-ming Science Tower**

Magda Stela REBELO

Modelling a competitive antibody/antigen reaction



Conference Programme — December 4 (Tue) —

09 : 00 – 09 : 50 **LT1, Cha Chi-ming Science Tower**

Stefan VANDEWALLE

Optimized overlapping Schwarz methods for parabolic PDEs with time-delay

09 : 55 – 10 : 45

Chengjian ZHANG

Unique solvability of numerical methods for delay-integro-differential equations

.....
Morning break
.....

11 : 10 – 12 : 00 **LT1, Cha Chi-ming Science Tower**

Ming-Zhu LIU

Numerical methods for impulsive ordinary differential equations

12 : 00 – 14 : 00 **Staff Dining Room, Sir Run Run Shaw Building**

.....
Lunch
.....

14 : 00 – 14 : 50 **LT1, Cha Chi-ming Science Tower**

Sui Sun CHENG

Closed form solutions of several types of differential equations

14 : 55 – 15 : 25

Yanping CHEN

Convergence analysis of the Jacobi collocation methods for Volterra integral equations with a weakly singular kernel

.....
Afternoon break
.....

15 : 55 – 16 : 25 **LT1, Cha Chi-ming Science Tower**

Wansheng WANG

Convergence of Runge-Kutta methods for neutral Volterra delay-integro-differential equations

16 : 30 – 17 : 00

Xiang XU

Spectral method for Volterra-type equation and postprocessing for initial-value problems

18 : 00 – **Renfrew Restaurant, Lam Woo International Conference Center**

.....
BANQUET
.....



Conference Programme — December 5 (Wed) —

09 : 00 – 09 : 50 **LT1, Cha Chi-ming Science Tower**

Tao TANG

Some recent developments on spectral methods for Volterra integral equations: convergence and stability

09 : 55 – 10 : 45

Emiko ISHIWATA

On collocation method for delay differential equations and Volterra integral equation with proportional delay

.....
Morning break
.....

11 : 10 – 12 : 00 **LT1, Cha Chi-ming Science Tower**

Hermann BRUNNER

Open problems in the numerical analysis of Volterra-type functional equations with vanishing delays

12 : 00 – 14 : 00 **Staff Dining Room, Sir Run Run Shaw Building**

.....
Lunch
.....

14 : 00 – **LT1, Cha Chi-ming Science Tower**

.....
Open problems and future work
.....

16 : 30 – **NTT (Pick up)**

.....
VICTORIA HARBOR TOUR
.....

~ 22 : 00 **NTT (Drop off)**



Conference Programme — December 6 (Thu) —

09 : 00 – 09 : 50 **LT1, Cha Chi-ming Science Tower**

Hongjiong TIAN

Dissipative dynamical systems and their numerical simulations

09 : 55 – 10 : 45

Yoshiaki MUROYA

Global stability for a class of discrete models

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CLOSING CEREMONY
.....

12 : 00– **Staff Dining Room, Sir Run Run Shaw Building**

.....
Lunch
.....



Contractivity and asymptotic stability properties of Runge-Kutta methods for Volterra functional differential equations

Shoufu LI
Xiangtan University, Xiangtan, China

A series of contractivity and asymptotic stability results of Runge-Kutta methods for contractive nonlinear Volterra functional differential equations (VFDEs) is obtained, some application of the results to nonlinear delay differential equations (DDEs) and delay integro-differential equations (DIDEs) are also presented. The results obtained in the present paper seem to be more general and deeper than the related existing results of Runge-Kutta methods for DDEs and DIDEs in literature.

For the convenience of the reader, list our main results as follows.

§1 Properties of the true solution $y(t)$ of the problem in VFDEs.

$$\begin{cases} y'(t) = f(t, y(t), y(\cdot)), & t \geq 0, \\ y(t) = \varphi(t), & t \leq 0 \end{cases} \quad (1)$$

satisfying the condition

$$\begin{cases} \langle f(t, u_1, \psi(\cdot)) - f(t, u_2, \psi(\cdot)), u_1 - u_2 \rangle \leq \alpha \|u_1 - u_2\|^2 \\ \quad \forall t \geq 0, u_1, u_2 \in \mathbf{R}^m, \psi \in \mathbf{C}_m(\mathbf{R}), \\ \|f(t, u, \psi_1(\cdot)) - f(t, u, \psi_2(\cdot))\| \leq \beta \max_{t-\mu_2(t) \leq \xi \leq t-\mu_1(t)} \|\psi_1(\xi) - \psi_2(\xi)\| \\ \quad \forall t \geq 0, u \in \mathbf{R}^m, \psi_1, \psi_2 \in \mathbf{C}_m(\mathbf{R}). \end{cases} \quad (2)$$

Theorem 1 $y(t)$ is stable on finite interval $[0, T]$ provided $\max\{\alpha + \beta, 0\}$ is of moderate size.

Theorem 2 $y(t)$ is contractive on the interval $[0, +\infty)$ provided $\alpha + \beta \leq 0$.

Theorem 3 $y(t)$ is strictly contractive and asymptotically stable on the interval $[0, +\infty)$ provided that $\alpha + \beta < 0$ and $\lim_{t \rightarrow +\infty} (t - \mu_2(t)) = +\infty$.

§2 Properties of numerical solution $\{y_n\}$ of Runge-Kutta method for VFDEs.

$$\begin{cases} y^h(t) = \Pi^h(t; \psi, y_1, y_2, \dots, y_{n+1}), & t \leq t_{n+1}, \\ Y^{(n+1)} = ey_n + h_n AF(Y^{(n+1)}, y^h(\cdot)), \\ y_{n+1} = y_n + h_n b^T F(Y^{(n+1)}, y^h(\cdot)), \end{cases} \quad (3)$$

which is always assumed to satisfy a canonical condition.

Theorem 4 If the method (3) is algebraically stable and diagonally stable, then this method is B -stable on finite interval $[0, T]$.

Theorem 5 Assume that the method (3) is algebraically stable, and there exist constants $c_1 > c_2 \geq 0$ which depend only on the method, such that for any two parallel integration steps $(t_n, \psi, y_1, \dots, y_{n+1}) \rightarrow$

$(t_{n+1}, \psi, y_1, \dots, y_{n+1}, Y_{n+1})$ and $(t_n, \chi, z_1, \dots, z_{n+1}) \rightarrow (t_{n+1}, \chi, z_1, \dots, z_{n+1}, Z_{n+1})$, we have

$$\langle W_{n+1}, BW_{n+1} \rangle \geq c_1 \|w_{n+1}\|^2 - c_2 \|w_n\|^2, \quad (4)$$

where $w_n = y_n - z_n$, $W_{n+1} = Y^{(n+1)} - Z^{(n+1)}$, $B = \text{diag}(b)$. Then the following are true:

- (1) the numerical solution $\{y_n\}$ is contractive on the interval $[0, +\infty)$ provided $\alpha + q\beta \leq 0$, where $q := \frac{c_\pi}{\sqrt{c_1 - c_2}}$, the constant c_π depends only on the interpolation operator Π^h ;
- (2) the numerical solution $\{y_n\}$ is strictly contractive and asymptotically stable on the interval $[0, +\infty)$ provided that $\alpha + q\beta < 0$ and $\lim_{t \rightarrow +\infty} (t - \mu_2(t)) = +\infty$.

§3 Efficient Runge-Kutta methods for VFDEs.

Theorem 6 All the s stage Radau IA, Radau IIA ($s \geq 1$) and Lobatto IIIC ($s \geq 2$) Runge-Kutta methods of the form (3) for VFDEs satisfy the assumption of Theorem 5. All the s ($s \geq 1$) stage Gauss, Radau IA, Radau IIA and the two stage Lobatto IIIC Runge-Kutta methods of the form (3) for VFDEs are B -stable.

§4 Application to DDEs, IDEs and DIDEs.

Delay-dependent stability of numerical methods for delay differential equations¹

Chengming HUANG

Huazhong University of Science and Technology, Wuhan, China

The stability of discretization methods plays an important role in the numerical solution of delay differential equations. One of the interesting problems in stability analysis is the investigation of the delay-dependent stability region of numerical methods.

A basic model problem in this field is the discrete delay equation

$$y'(t) = \alpha y(t) + \beta y(t - \tau)$$

where $\alpha, \beta \in \mathbb{R}$ and τ is a fixed positive number. A very large body of results have been obtained for this equation. In particular, Guglielmi and Hairer (*Numer. Math.*, 83(1999), 371-383.) developed the relationship between stability analysis and order stars, which leads to an elegant result that all Gauss methods are $\tau(0)$ -stable. In addition, they also proved that the s -stage Radau IIA methods are $\tau(0)$ -stable for $s = 2, 3$ and obtained the numerical evidence that Radau IIA methods are $\tau(0)$ -stable for $4 \leq s \leq 7$. Therefore, it is still an open problem whether all Radau methods are $\tau(0)$ -stable.

In this talk, a new theoretical framework will be given to study delay-dependent stability for numerical methods. As an application of the framework, a class of Runge-Kutta methods of high order, which has as special cases all Radau and Gauss methods, are proven to be $\tau(0)$ -stable.

¹Supported by the National Natural Science Foundation of China (Grant No.10671078) and by the Program for NCET, the State Education Ministry of China.

Stability of IMEX methods for ordinary and delay differential equations

Toshiyuki KOTO
Nagoya University, Japan

Let us consider ordinary differential equations of the form

$$\frac{du}{dt} = Lu(t) + g\left(t, u(t)\right), \quad (1)$$

where $u(t)$ is a vector valued unknown function and L is a square matrix. We suppose that $Lu(t)$ on the right hand side gives a stiff term. A typical example of such equations arises after the spatial discretization of a partial differential equation of reaction diffusion or convection diffusion type, and some special numerical methods for solving (1) have been proposed along the idea of treating the linear stiff term by an implicit scheme and the nonlinear term by an explicit scheme. Such a method, called IMEX (implicit explicit) method, can be also applied to delay differential equations of the form

$$\frac{du}{dt} = Lu(t) + g\left(t, u(t), u(t - \tau)\right), \quad (2)$$

where $\tau > 0$ is a constant delay.

Recently, we have studied stability of IMEX Runge Kutta methods using linear test equations [1, 2]. We here carry out similar analyses of IMEX linear multistep methods. This is a joint work with Yuka Hiraide (Graduate School of Information Science, Nagoya University).

References:

- [1] T. Koto, Stability of IMEX Runge Kutta methods for delay differential equations, J. Comput. Appl. Math., in press.
- [2] T. Koto, IMEX Runge Kutta schemes for reaction diffusion equations, J. Comput. Appl. Math., in press.

Application of PPMLR scheme to the numerical simulation of fluid instability in inertial confinement fusion

Xiaomei QU
Xiangtan University, China

This paper is devoted to the study of high order Godunov scheme PPMLR and its application to the numerical simulation of fluid instability problems which appear frequently in Inertial Confinement Fusion (ICF). At first, we improve the PPMLR scheme presented by Colella and Woodward in two aspects: (1) the flattening algorithm for treating shocks is appropriately modified, (2) a new procedure for treating the dependent variables obtained from the remapping is given. Then we applied the improved PPMLR scheme to the numerical simulation of Richtmyer-Meshkov instability, Rayleigh-Taylor instability problems and computation of compressible multi-component large distortion flows, satisfied results are obtained. Lots of numerical experiments indicate that, the flexibility of the improved PPMLR scheme is very good, even if for high density ratio flow problems, multi-component strong shock problems and multi-component large distortion vortex problems, the numerical results are satisfied. Finally, we have compared the PPMLR scheme, the fifth-order FD-WEND scheme and the MUSCL scheme quantitatively, and find that PPMLR scheme and fifth-order FD-WEND scheme both have their own advantages; are outstanding schemes and can be chosen for ICF numerical simulation, but MUSCL scheme looks somewhat inferior, and we thus do not suggest it.

A new approach to the numerical solution of forward-backward equations

Filomena TEODORO

Instituto Superior Técnico, Lisbon and Instituto Politécnico de Setúbal, Setúbal, Portugal

P. M. LIMA,

Instituto Superior Técnico, Lisbon, Portugal

N. J. FORD and P. M. LUMB

University of Chester, UK

This talk is concerned with the analysis of forward-backward equations of the general form:

$$x'(t) = a(t)x(t) + b(t)x(t-1) + c(t)x(t+1). \quad (1)$$

We search for a solution $x(t)$, defined on a certain interval $[-1, k]$, ($k > 0$), which satisfies the following conditions:

$$x(t) = \begin{cases} \Phi_1(t), & t \in [-1, 0]; \\ f(t), & t \in (k-1, k], \end{cases} \quad (2)$$

where Φ_1 and f are given functions.

This problem has been studied both analytically and numerically (see[1]).

One of the most common approaches for the analysis of this problem is based on its reduction to an initial value problem for a delay differential equation (DDE).

Following this, we search for an approximate solution in the form

$$x_N(t) = x_0(t) + \sum_{j=1}^{N-1} C_j x_j(t), \quad t \in [-1, k] \quad (3)$$

where x_0 is an initial approximation of the solution, $\{x_j\}_{1 \leq j \leq n}$ is a set of *basis functions*.

The approximate solution can be extended to the interval $[1, k]$ either numerically (using the finite difference method) or analytically (using the recurrence formulae - “method of steps”).

Finally, the coefficients c_j of the expansion (3) are computed by the collocation or least squares method, so that $x^{(N)}(t)$ approximates $f(t)$, as $t \in [k - 1, k]$.

Numerical results obtained by this method are presented and compared with the ones, presented in previous works.

The advantages and weaknesses of the introduced computational method are discussed.

Reference:

[1] N. Ford and P. Lumb, Mixed type functional differential equation: a numerical approach (submitted).

Modelling a competitive antibody/antigen reaction

Magda Stela REBELO, Teresa DIOGO
Instituto Superior Técnico, Lisbon, Portugal

Seam McKEE
University of Strathclyde, UK

This work is concerned with modelling the evolution of competitive chemical reactions within a small cell with a labelled and unlabelled antigen reacting with a specific antibody on the side wall. A model consisting of coupled heat conduction equations with nonlinear and nonlocal boundary conditions is considered and shown to be equivalent to a system of Volterra integral equations with weakly singular kernel. This work generalizes some previous work done on the case of the single heat equation ([1], [2]). We prove the existence and uniqueness of the nonlinear system on $[0, \infty]$. The asymptotic behavior of the solution as $t \rightarrow 0$ and $t \rightarrow \infty$ is obtained and other properties of the solution, e.g., monotonicity, are investigated. In order to obtain a numerical solution of the system of VIES we use the technique of subtracting out singularities to derive explicit and implicit Euler schemes with order one convergence and a product trapezoidal scheme with order two convergence. Numerical results are presented.

References:

[1] S.Jones, B.Jumarhon, S.McKee, J.A.Scott *A mathematical model of a biosensor*, Journal of Engineering Mathematics 30, Netherlands, (1996) 321-337.

[2] B.Jumarhon, S.McKee *On the heat equation with nonlinear an nonlocal boundary conditions*, Journal of Mathematical Analysis and Applications 190, (1995) 806-820.



Optimized overlapping Schwarz methods for parabolic PDEs with time-delay

Stefan VANDEWALLE
Katholieke Universiteit Leuven, Belgium

Parabolic delay partial differential equations model physical systems for which the evolution does not only depend on the present state of the system but also on the past history. Such models are found, for example, in population dynamics and epidemiology, where the delay is due to a gestation or maturation period, or in numerical control, where the delay arises from the processing in the controller feedback loop. In the first part of the talk we will study the analytical properties of the solutions of parabolic delay PDEs. Two model problems will be considered in particular: the heat equation with a fixed delay term, and the heat equation with a distributed delay in the form of an integral over the past. It will be shown that the dynamics of delay PDEs is fundamentally different from that of regular time-dependent PDEs without time delay.

Next, we will study the numerical solution of the above model problems with overlapping Schwarz methods. The considered methods are of waveform relaxation type: they compute the local solution in each subdomain over many timelevels before exchanging boundary information to neighbouring subdomains. We analyse the effect of the overlap width and we derive optimized transmission boundary conditions of Robin type. The theoretical results and convergence estimates are verified through some numerical experiments.

Unique solvability of numerical methods for delay-integro-differential equations

Chengjian ZHANG
Huazhong University of Science and Technology, Wuhan, China

Delay-integro-differential equations (DIDEs) arise widely in the mathematical modeling of physical and biological phenomena. Significant advances in the research of theoretical solutions and numerical solutions for such equations have been made in recent years (see e.g. [1, 2, 3]). A survey of the related results can be referred to Brunner (2004)'s book (cf. [4]). Among these existing research, ones devote themselves to studying stability, convergence and computational implement of the methods. When numerically computing a DIDE, generally to speak, an algebraic equation need to be solved. What is the condition to guaranteed the unique solvability of the algebraic equation? This is an important problem for numerical computation of DIDEs. However, so far, very few references deal with such a topic except those for standard ODEs. In view of this, in the presented talk we will introduce our findings on this topic. The unique solvability conditions of the extended linear multistep methods, Runge-Kutta methods and general linear methods for DIDEs will be concerned.

References:

- [1] C. Zhang and S. Vandewalle, Stability analysis of delay-integro-differential equations and their

backward differentiation time-discretization, J. Comp. Appl. Math, Vol. 164-165, pp. 797-814

[2] C. Zhang and S. Vandewalle, Stability Analysis of Runge-Kutta Methods for Nonlinear Volterra Delay-integro-differential Equations, IMA Numer. Anal., 24 (2004), pp. 193-214.

[3] C. Zhang and S. Vandewalle, General Linear Methods for Volterra Integro-differential Equations with Memory, SIAM J. Sci. Comput., 27 (2006), pp. 2010-2031.

[4] H. Brunner, Collection Methods for Volterra Integral and related Functional Equations, Cambridge University Press, Cambridge, 2004.

Numerical methods for impulsive ordinary differential equations

Ming-Zhu LIU, H. LIANG and X. J. RAN
Harbin Institute of Technology, Harbin, China

In this talk we will discuss the numerical methods for impulsive ordinary differential equations (IODEs)

$$\begin{cases} \dot{x} = f(t, x(t)), & t \neq \tau_k, \quad t > 0, \\ \Delta x = I_k(x), & t = \tau_k \\ x(0^+) = x_0, \end{cases} \quad (1)$$

where $x : \mathbb{R}^+ \rightarrow \mathbb{C}^d$, $f, I_k (k = 0, 1, \dots)$ are continuous functions, $\Delta x = x(t^+) - x(t)$, $x(t^+)$ is the right limit of $x(t)$, $0 = \tau_{-1} < \tau_0 < \tau_1 < \dots$, and $\lim_{k \rightarrow \infty} \tau_k = \infty$.

Firstly, a surprising example is given. The numerical methods exhibit the quite different stability properties when the methods apply to IODEs and ODEs. For example, the implicit Euler method can not preserve the stability of linear IODEs with constant coefficients, while the explicit Euler method can preserve that.

For (1), we construct a scheme of Runge Kutta methods. It is shown that the numerical solutions converge to the analytic solutions at accuracy of the original order p of the method, provided that $f \in C^P$ and $I_k, k = 1, 2, \dots$, satisfies the Lipschitz conditions.

At last, the conditions under which the Runge Kutta methods preserve the stability property of IODEs are obtained and some numerical experiments are given.

Closed form solutions of several types of differential equations

Sui Sun CHENG
National Tsing Hua University, Taiwan

When confronting an unfamiliar differential equation, it is natural to find the simplest type of solutions such as polynomial, rational, exponential, power and analytic solutions. I will describe, by means of simple examples, several types of differential equations which admits closed form solutions and how they can be computed.

Convergence analysis of the Jacobi collocation methods for Volterra integral equations with a weakly singular kernel

Yanping CHEN

Xiangtan University, China

Tao TANG

Hong Kong Baptist University, Hong Kong

In this paper, a Jacobi-collocation spectral method is developed for Volterra integral equations of second kind with weakly singular kernel. We use some function transformation and variable transformations to change the equation into a new Volterra integral equation defined on the standard interval $[-1, 1]$, so that the solution of the new equation possesses better regularity and Jacobi orthogonal polynomials theory can be applied conveniently. In order to obtain high order accuracy for the approximation, the integral term in the resulting equation is approximated by Gauss quadrature formula using the Jacobi collocation points and Jacobi weight. The convergence analysis of this novel method is based on the Lebesgue constants corresponding to the Lagrange interpolation polynomials, polynomials approximation theory for orthogonal polynomials, the operator theory, and some other important inequalities. The spectral rate of convergence for the proposed method is established in the L^∞ -norm and weighted L^∞ -norm. Numerical results are presented to demonstrate the effectiveness of the proposed method.

Convergence of Runge-Kutta methods for neutral Volterra delay-integro-differential equations

Wansheng WANG

Xiangtan University and Changsha UST, China

Shoufu LI

Xiangtan University, China

This paper is concerned with the numerical solution of neutral Volterra delay-integro-differential equations (NVDIDEs) with constant delay $\tau \geq 0$,

$$y'(t) = f\left(t, y(t), y(t - \tau), \int_{t-\tau}^t K(t, \theta, y(\theta), y'(\theta))d\theta\right), \quad t \geq 0,$$

subject to $y(t) = \phi(t)$, $t \in [-\tau, 0]$. We focus on the error behaviour of Runge-Kutta methods for stiff NVDIDEs. Convergence properties of Runge-Kutta methods are investigated. The convergence results obtained in this paper provide unified theoretical foundation for the convergence analysis of solutions to nonlinear stiff problems in delay differential equations (DDEs), Volterra delay integro-differential equations (VDIDEs) and neutral Volterra delay integro-differential equations (NVDIDEs) of other type which appear in practice.

Spectral method for Volterra-type equation and postprocessing for initial-value problems

Xiang XU

Fudan University, China

Tao TANG

Hong Kong Baptist University, Hong Kong

Spectral method has been widely used for its infinite order in accuracy. But most of the work was done for boundary value problem. In the following, a Legendre-collocation method is proposed to solve the Volterra integral equations of the second kind. We also provide a rigorous error analysis for the proposed method, which indicate that the numerical errors (in the infinity norm) will decay exponentially provided that the kernel function and the source function are sufficiently smooth. For the volterra type equation behaviors like an initial problem, from the method we can get a post-processing method to improve the accuracy of some lower order method for initial value problems. Numerical results shows that the method for initial value problems are efficiently.



Some recent developments on spectral methods for Volterra integral equations: convergence and stability

Tao TANG
Hong Kong Baptist University, Hong Kong

In this talk, we will review some relevant theory of spectral methods which is useful for studying the convergence property of spectral methods to Volterra integral equations. Some numerical methods will be proposed, and their convergence analysis will be provided. It will be shown that the numerical errors will decay exponentially provided that the kernel function and the source function are sufficiently smooth. Some ideas on stability analysis will be also provided.

On collocation method for delay differential equations and Volterra integral equation with proportional delay

Emiko ISHIWATA
Tokyo University of Science, Japan

We consider the pantograph differential equation with proportional delay qt , $0 < q \leq 1$:

$$y'(t) = ay(t) + by(qt) + f(t), \quad y(0) = y_0. \quad (1)$$

For delay differential equation, Bellen [1] and Bellen *et al.* [2] recently showed results for superconvergence in numerical methods, and for the Volterra integral equation $y(t) = f(t) + \int_0^t k_1(t, s, y(s))ds + \int_0^{qt} k_2(t, s, y(s))ds$, $t \in J := [0, T]$ with proportional delay qt , $0 < q \leq 1$, Brunner and Hu [3] established a general condition characterizing meshes for which local super-convergence order at the nodes can occur, provided that the collocation parameters are the m Gauss points in $[0, 1]$.

In this talk, for (1), we offer practical “quasi-constrained meshes” which preserve such optimal superconvergence at the nodes of the underlying mesh and consider global error analysis of successive mesh points (cf. [4]). This mesh is useful in case that a long time integration is needed, because the maximal mesh size of this method is not greater than the first mesh size, and number of steps in our method is less than that of the collocation method. Related topics for attainable order of collocation method are also contained (see for example, [5] and references therein).

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Open problems in the numerical analysis of Volterra-type functional equations with vanishing delays

Hermann BRUNNER

Memorial University of Newfoundland, Canada / Hong Kong Baptist University

The years 1971 and 1992/93 saw, respectively, the introduction of the pantograph equation, $y'(t) = ay(t) + by(qt)$ ($t \geq 0, 0 < q < 1$), and the beginning of its in-depth numerical analysis. It is perhaps not as widely known that the very first studies of integral equations with proportional delay qt date back to the work of Volterra (1897/1913), Picard (1907), Lalesco (1908/1911), and Andreoli (1913/1914). While the (quantitative) theory of pantograph-type delay differential and integral equations is now largely understood, this is not true of their numerical analysis.

This talk will start with a survey of some recent advances in the analysis of superconvergence of collocation solutions for functional integral and integro-differential equations with vanishing delays. It will then focus on the underlying analytical and computational difficulties inherent in these “innocent-looking” delay equations and show that many questions remain unanswered. In particular, it is not known under which conditions collocation methods on uniform meshes yield uniformly convergent solutions to the oldest of these functional integral equations, namely Volterra’s first-kind equation of 1897. The presence of weakly singular kernels in these functional equations leads to additional complexities in the numerical analysis. Also, much of the stability analysis of numerical solutions to functional integral and integro-differential equations on uniform meshes remains open, even for the linear vanishing delay $\theta(t) = qt$ ($0 < q < 1$).



Dissipative dynamical systems and their numerical simulations

Hongjiong TIAN

Shanghai Normal University, Shanghai, China

Many problems arising in physics and engineering are modeled by dissipative differential equations where an energy loss mechanism is present. Dissipative differential equations are featured by possessing a bounded positively invariant absorbing set which all trajectories starting from any bounded set enter in a finite time and thereafter remain inside.

Runge-Kutta and linear multistep methods are often used to obtain a numerical solution of ordinary differential equations or delay differential equations. A numerical method convergent in a finite interval does not necessarily yield the same asymptotic behavior as the underlying differential equation. It is desirable to design numerical schemes for which the limit sets are close to the corresponding ones for the underlying differential equation, and to understand and hence to avoid conditions under which spurious members of the limit sets are introduced by the time discretisation.

This talk contains three parts:

1. Dissipativity of ordinary differential equations and their numerical methods,
2. Dissipativity of delay differential equations and their numerical methods, and
3. Dissipativity of delay functional differential equations and one-leg θ -method.

Global stability for a class of discrete models

Yoshiaki MUROYA

Waseda University, Tokyo, Japan

Usually, the convergence of the iterations for the discretized models derived from some nonlinear problems is dependent to the initial data (“uniform asymptotic stability”, see [1-5]) and is very important in numerical analysis. This time, we are interested in special cases that the convergence is independent of the initial data (“global asymptotic stability”), for example, such cases occur in the homotopy method.

In this talk, using “semi contractivity” of functions, we investigate sufficient conditions for the zero solution of the following nonlinear difference equation to be globally asymptotically stable (cf. [6-8]).

$$\begin{cases} x_{n+1} = qx_n + (1-q) \sum_{i=1}^m b_{n,i} g_i(x_{n-i}), & n = 0, 1, 2, \dots, \\ x_j = \phi_j, & -m \leq j \leq 0, \end{cases}$$

where $g_i(x)$ ($i = 0, 1, \dots, m$) are suitable functions.

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Victoria Harbor Tour — December 5 (Wed) —

Pick up at NTT at 4:30pm

Kowloon Public Pier (TST)

Victoria Harbour → Aberdeen

The Aberdeen Fishing Village

Aberdeen → Sok Kwu Wan

Rainbow Gold Prize Seafood Restaurant Seafood Dinner

1. Fried Crab with Honey & Pepper Sauces
2. Boiled Prawn
3. Fried Lobster with Butter Sauce
4. Deep Fried Squid Cutlet
5. Steamed Scallop with Mashed Garlic
6. Sweet and Sour pork
7. Boiled Seasonal Vegetable
8. Lamma Island fried Rice with Shrimp Paste
9. Seasonal Fruit
10. Chinese Tea

Sok Kwu Wan → Victoria Harbour

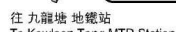
Cruise of *The Pearl of Orient* night view

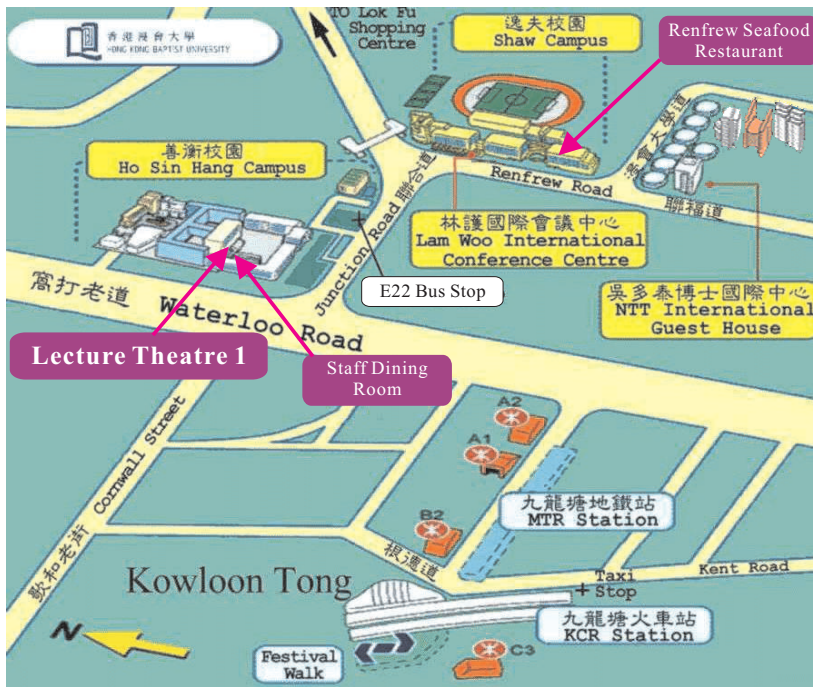
Symphony of Lights Show

To the top of Hong Kong by tram

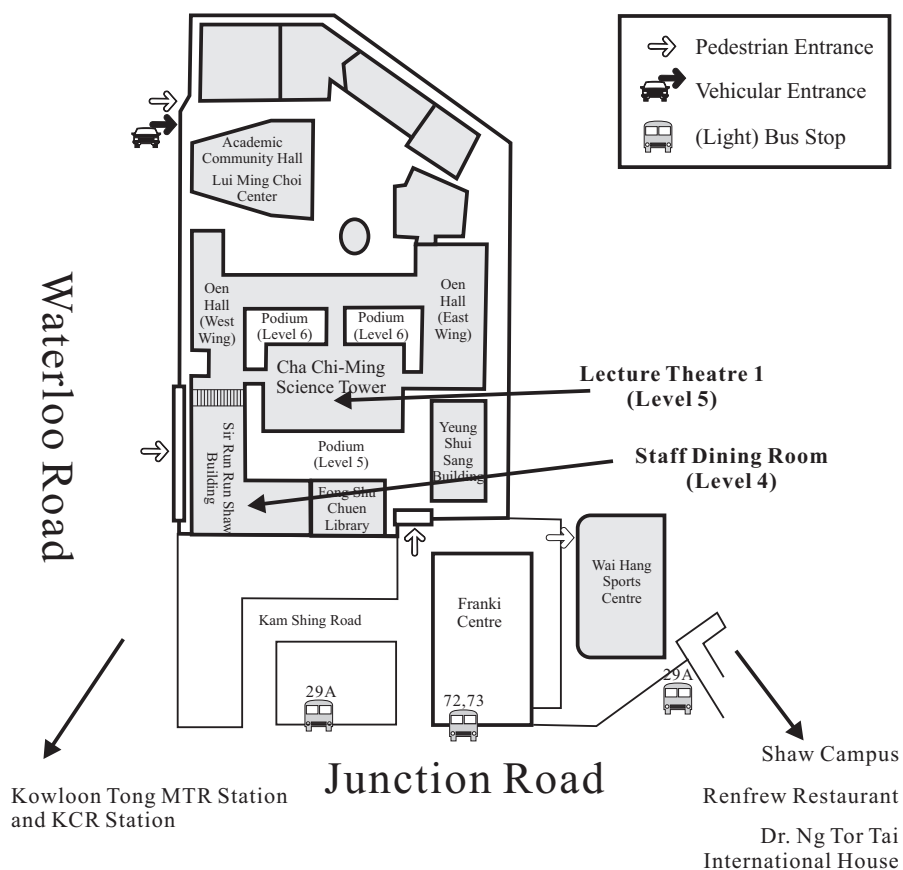
Walk through Lan Kwai Fong

Back to Hotel at 10:00pm





Ho Sin Hang Campus





List of Participants

Brunner, Hermann
Memorial University of Newfoundland, Canada.
hbrunner@math.hkbu.edu.hk

Cheng, Sui Sun
Tsing Hua University, Taiwan, Taiwan.
sscheng@math.nthu.edu.tw

Diogo, Teresa
Instituto Superior Técnico, Portugal.
tdiogo@math.ist.utl.pt

Huang, Chengming
Huazhong University of Science and Technology, China.
chengming_huang@hotmail.com

Koto, Toshiyuki
Nagoya University, Japan. *koto@is.nagoya-u.ac.jp*

Li, Congying
Xiangtan University, China. *quxiaomei123@163.com*

Lin, Yinwei
National Sun Yat-sen University, Taiwan.
timothy@math.nsysu.edu.tw

Ling, Leevan
Hong Kong Baptist University, Hong Kong.
lling@hkbu.edu.hk

Lu, Shannian
Higher Education Press, China. *lushn@hep.com.cn*

Muroya, Yoshiaki
Waseda University, Japan. *ymuroya@waseda.jp*

Noborisaka, Chihiro
Waseda University, Japan. *momora0826@msn.com*

Rebelo, Magda
Instituto Superior Técnico, Portugal. *msjr@fct.unl.pt*

Teodoro, Maria
Escola Superior Tecnologia de Setúbal, Portugal.
mteodoro@est.ips.pt

Chen, Yanping
Xiangtan University, China. *ypchen@xtu.edu.cn*

Ding, Haijia
Higher Education Press, China. *dinghj@hep.com.cn*

Duan, Jinqiao
Illinois Institute of Technology, USA. *duan@iit.edu*

Ishiwata, Emiko
Tokyo University of Science, Japan.
ishiwata@rs.kagu.tus.ac.jp

Kuroda, Masataka
Waseda University, Japan. *bannyuaai@fuji.waseda.jp*

Li, Shoufu
Xiangtan University, China. *lisf@xtu.edu.cn*

Ling, Feng
Zhaoqing University, China. *lingf@zqu.edu.cn*

Liu, Ming-Zhu
Harbin Institute of Technology, China. *mzliu@hit.edu.cn*

Mitsuda, Ken-Ichiro
Nagoya University, Japan. *mitsuda@math.cm.is.nagoya-u.ac.jp*

Nakata, Yukihiro
Waseda University, Japan. *yunayuna.na@gmail.com*

Qu, Xiaomei
Xiangtan University, China. *maggiequ111@163.com*

Tang, Tao
Hong Kong Baptist University, Hong Kong.
ttang@hkbu.edu.hk

Tian, Hong Jiong
Shanghai Normal University, China.
hjtian@shnu.edu.cn

Tsai, Arthur
Academia Sinica, Taiwan. *arthur@stat.sinica.edu.tw*

Wan, Zhengsu
Hunan Institute of Science and Technology, China.
zs77w@126.com

Wang, Wansheng
Xiangtan University, China. *w.s.wang@163.com*

Wen, Liping
Xiangtan University, China. *lpwen@xtu.edu.cn*

Yang, Shuiping
Xiangtan University, China. *yang52053052@sohu.com*

Zhao, Tao
Chinese Academy of Sciences, China.
tzhou@lsec.cc.ac.cn

Vandewalle, Stefan
University of Leuven, Belgium.
stefan.vandewalle@cs.kuleuven.be

Wang, Heyu
Hong Kong Baptist University, Hong Kong.
hywang@math.hkbu.edu.hk

Wang, Wenqiang
Xiangtan University, China. *wwq@xtu.edu.cn*

Xu, Xiang
Fudan University, China. *xuxiang@fudan.edu.cn*

Zhang, Chengjian
Huazhong University of Science and Technology, China.
cjzhang@hust.edu.cn