

# On the Sandpile Group of a Graph

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The *abelian sandpile model* (ASM) is a prime example of *self-organized criticality* which has transformed the understanding of how complexity arises in nature by Dhar. The ASM is related to the chip-fire game, introduced and discussed by Spencer, Björner, Lovász and Biggs. The sandpile group of a graph  $G$  is closely connected with the Laplacian matrix  $L(G)$  as follows: Thinking of  $L(G)$  as a linear map  $\mathbb{Z}^n \rightarrow \mathbb{Z}^n$ , its *cokernel* has the form  $\text{coker } L(G) = \mathbb{Z}^n / L(G)\mathbb{Z}^n \cong \mathbb{Z} \oplus S(G)$ , where  $S(G)$  is the *sandpile group* on  $G$  in the sense of isomorphism.

In this talk, the problem for determining the concrete and abstract structure of the sandpile groups of some graphs is discussed.