# Algebraic Approach on Balance Index Sets of Graphs

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Let $G = (V, E)$ be a simple graph. A labeling $f : V \to A = \{0, 1\}$ induces a partial edge labeling $f^* : E \to A$ defined by $f^*(uv) = f(u)$ if and only if $f(u) = f(v)$ for each edge $uv \in E$, other edges are not labeled. For each $i \in A$, let $v_f(i) = |f^{-1}(i)|$ and $e_f(i) = |f^{*-1}(i)|$. $f$ is called a friendly labeling if $|v_f(1) - v_f(0)| \leq 1$. A graph is called balanced if there is a friendly labeling $f$ such that $|e_f(0) - e_f(1)| \leq 1$. Such labeling $f$ is called a balanced labeling of $G$.

For a friendly labeling of a graph $G$, $B_f(G) = e_f(1) - e_f(0)$ is called the balance index of $G$ under $f$. The sets

$$\mathrm{BI}(G) = \{|B_f(G)| \mid f \text{ is friendly}\}.$$
$$\mathrm{FBI}(G) = \{B_f(G) \mid f \text{ is friendly}\}.$$

are called balance index set and full balance index set of $G$, respectively.

In this talk, I will define a new equivalent definition for balance index. Also I will provide new proofs for some existing results by using this new approach. Some balance index sets of graphs will be found.