Computational Approach To Solvability Of Refinement Equations

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The solvability and Fredholm properties of refinement equations in spaces of square-integrable functions are studied. Necessary and jointly necessary and sufficient conditions for the solvability of homogeneous and non-homogeneous refinement equations are established. It is shown that in the space $L_2(\mathbb{R})$ the kernel space of any homogeneous equation with a non-trivial solution is infinite dimensional. Moreover, the solvability problem is reduced to the study of singular values of certain matrix sequences. These sequences arise from Galerkin approximations of auxiliary linear operators. The corresponding constructions use only the coefficients of refinement equations that generate multiresolution analysis, and the coefficients of the refinement equation studied. For the equations with polynomial symbols the most complete results are obtained if the corresponding operator is considered on an appropriate subspace of the space $L_2(\mathbb{R})$.