Mixed Finite Element Methods For General Quadrilateral Grids

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In this talk, we show some peculiar aspects of mixed finite elements for general quadrilateral grids for elliptic problems. As is typical with all finite element methods, all the functions are defined on a reference element and then mapped onto the general element via certain mappings. Scalar functions are mapped in the usual way, by composition. Meanwhile, vector functions are mapped by a special map, called Piola map to preserve divergence of the function. This does not cause much difficulties in case of triangular grids. However, some problems arise when quadrilateral grids are used: First, the divergence of approximate velocity space loses optimal approximation property due to the lack of proper polynomials: Second, the divergence of vector fields no longer lie in the pressure(scalar) field, which causes stability problem.

Based on the above observation, we suggest some new element which is a modification of Raviart-Thomas element of lowest order. This new element is designed so that the divergence of velocity fields lie in the pressure space, and $\mathbf{H}(\text{div})$ -projection Π_h satisfies div $\cdot \Pi_h = P_h$ div. A rigorous optimal order error estimate is carried out by proving a modified version of the Bramble-Hilbert lemma for vector variables. We show a local $\mathbf{H}(\text{div})$ projection reproducing certain polynomials suffices to yield an optimal \mathbf{L}^2 error estimate for the velocity and hence our approach also provides an improved error estimate for original Raviart-Thomas element of lowest order.