## Error Estimates For Degenerate Quadrilateral Finite Elements

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In this paper, we study the relation between the error estimate of the bilinear interpolation on a general quadrilateral and the geometric characters of the quadrilateral. Some explicit bounds of the interpolation error are obtained based on some sharp estimates of the integral  $\frac{1}{|J|^{p-1}}$  for  $1 \le p \le \infty$ on the reference element, where J is the Jacobian of the non-affine mapping. This allows us to introduce a weak geometric condition, which admits the optimal interpolation error estimate in  $W^{1,p}$  norm and can be regarded as a generalization of the RDP (regular decomposition property) condition. One of the main features of our estimates is that, if choose one diagonal and divide the element into two triangles, with the big triangle is always denoted by  $T_1$ and the other be  $T_3$ , the term  $\frac{|T_3|}{|T_1|}$  is also considered as an impact factor in the error bounds. As far as we know, the GRDP condition presented in this paper is weaker than any other mesh conditions proposed in the literatures for the same p with  $1 \le p \le \infty$ . The motivation of this paper comes from the idea that, when a quadrilateral is almost degenerated into a triangle, the bilinear interpolation error on the quadrilateral may be dominated by the linear interpolation error on the triangle.