A Chaotic Motion In The N-Vortex Problem On Sphere - Hamiltonian Systems With Saddle-Center Equilibria

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We study the motion of N point vortices with $N \in \mathbb{N}$ on a sphere in the presence of fixed pole vortices. It is written as a Hamiltonian dynamical system with N degrees of freedom. In this presentation, we focus on the evolution of their polygonal ring configuration called the *N-ring*, in which they are equally spaced along a line of latitude of the sphere when it is unstable. For $N = 5n$ or $6n$ with $n \in \mathbb{N}$, the system is reduced to a two-degree-of-freedom Hamiltonian with some saddle-center equilibria, one of which corresponds to the unstable N-ring. With a Melnikov-type method applicable to two-degree-of-freedom Hamiltonian systems with saddle-center equilibria and a numerical method to compute stable and unstable manifolds, we show that there exist transverse homoclinic orbits to unstable periodic orbits in the neighborhood of the saddle-centers by numerical means. Thus, the evolution of the unstable N-ring becomes chaotic.

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