Numerical Approximations To Hadamard Finite-Part Integral Operators

Weiwei Sun
City University of Hong Kong, Hong Kong
maweiw@math.cityu.edu.hk

Many physical problems require efficient discrete schemes for Hadamard finite-part integral operators and efficient quadrature rules for the evaluation of such integrals in the form

$$\oint_a^b \frac{u(x')}{(x'-x)^{p+1}}, dx' = g(x), \quad x \in (a,b), \quad p \geq 1 \tag{1}$$

where $s \in (a,\ b)$ is the singular point.

In this talk, we present a general framework of the interpolative quadrature rules for Hadamard finite-part integrals. The Gaussian quadrature rules and Newton-Cotes formulas are viewed as special cases. We show the pointwise superconvergence phenomenon of these interpolative quadrature rules, that is, when the singular point coincides with certain *a priori* known points, the accuracy is better than what is globally possible. A new quadrature rule of Gaussian type is proposed for the evaluation of integrals simultaneously with different types of singularities. Several collocation-type approximations to Hadamard integral operators are presented. These discrete schemes are of Toeplitz or nearly Toeplitz structure, which gives many advantages in developing fast linear solvers for numerical solution of integral equations and intego-differential equations.