On A Property Of Laplace Transforms Of Some Probability Densities

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It is known that all of probability distributions with density of normed product of the Cauchy densities such as

$$f(a,b;x) = \frac{c}{(a^2+x^2)(b^2+x^2)},\ (0<a<b)$$

are infinitely divisible. But it seems that it is not known if a probability distriution with density of normed product of the multi-dimensional Cauchy densities is infinitely divisible or not. In this talk we show a conjecture on the infinite divisiblity of some probability distributions with density of normed product of odd dimensional Cauchy densities, namely,

$$f(a,b;x) = \frac{c}{(a^2+|x|^2)^{(d+1)/2}(b^2+|x|^2)^{(d+1)/2}}, \tag{1}$$

where c is a normalised constant and

$$0<a<b; \quad x=(x_1,x_2,\cdots,x_d)\in \mathbf{R}^d.$$

In this talk we assume the dimension d is an odd integer. We should note that the density $f(a,b;x)$ can not be decomposed to a sum of partial fractions in the same way as in the 1-dimensional case and the speaker shows that we can overcome this difficulty. Making use of the formula

$$K_\nu(z) = \frac{1}{2}(\frac{1}{2}z)^\nu \int_0^\infty \exp\{-t-\frac{z^2}{4t}\}\,\frac{dt}{t^{\nu+1}}, \tag{2}$$

we obtain a Laplace-Stieltjes transform for the general odd dimensional case,

$$\zeta(d;s) = \frac{c\pi^{d/2}}{\{(b^2-a^2)^{l+1}}\Sigma_{j=0}^{l}\frac{(-1)^{2l-j}(l+j)!}{(b^2-a^2)^j}\binom{l}{j}2^{j+1/2}\cdot$$
$$\Big\{a^{2j-1}\frac{K_{(j-1)+1/2}(2a\sqrt{s})}{(2a\sqrt{s})^{j-1+1/2}}+(-1)^{l+1+j}b^{2j-1}\frac{K_{(j-1)+1/2}(2b\sqrt{s})}{(2b\sqrt{s})^{j-1+1/2}}\Big\}. \tag{3}$$