Iterative Behavior Of Symmetric Weighted Threshold Filter For Bounded Sequences With Infinite Support

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The symmetric weighted median filter is a natural extension of the median filter and has the same advantages as the median filter: edge preservation and efficient suppression of impulsive noise [1]. The symmetric weighted threshold filter is a generalization of the symmetric weighted median filter. It is natural to iterate the symmetric weighted threshold filter in order to reduce the noise still further. An interesting question to be asked is what iterative behavior the the symmetric weighted threshold filter has if we apply iteratively the symmetric weighted threshold filter to a fixed sequence. For sequences with finite support, it was shown that the iterated sequences of the symmetric threshold automaton converges or gets trapped in a periodic orbit of period 2 ([2,3]). P. D. Wendt [4] applied the fact to analyze the iterative behavior of stack filters and proposed a method to transform a symmetric weighted threshold filter into a symmetric threshold automaton if this symmetric weighted threshold filter preserves roots of median filter. But for sequences with infinite the method above does not work. In this paper, for bounded sequences with infinite support, we give a thorough answer to the question. On one hand, in our result we cancel the restrictive condition for symmetric weighted threshold filter to preserve roots of median filter. On the other hand, a sequence with finite support may be transferred into a sequence with infinite support by appending two ends of the original sequence or by expending periodically the original sequence. Specifically, for the original sequence with finite support $x_L = \{x_L(1), x_L(2), \cdots, x_L(L)\}$, the corresponding sequence with infinite support $x' = \{x'(n)\}_{n \in Z}$ is defined either by

$$x'(n) = \begin{cases} x(n), & \text{for } 1 \le n \le L \\ x(L), & \text{for } n > L \\ x(1), & \text{for } n < 1 \end{cases} \qquad n \in Z$$

or by

$$x^{(1)}(n) = \begin{cases} x(n), & \text{for } 1 \le n \le L \\ x(p), & \text{for } n = mL + p, m \in Z, 1 \le p \le L, \end{cases} \qquad n \in Z.$$

Therefore, our result is a generation of the above result.

References

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