Real-valued Iterative Algorithms for Complex Symmetric Linear Systems

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Large and sparse complex valued linear systems of algebraic equations arise in many applications. Sometimes the underlying matrices are *complex symmetric*, i.e., non Hermitian but have a symmetric pattern. A typical example for this category is the complex Helmholtz equation, time-dependent Schrödinger equation, inverse scattering problems, generalized eigenvalue problems and many others.

Let us write the underlying complex linear system in the form

$$(A + \mathbf{i}B)(x + \mathbf{i}y) = (b + \mathbf{i}c).$$

Complex symmetric linear systems will have A and B real symmetric matrices, which are assumed here to be large and sparse and $\mathbf{i} = \sqrt{-1}$. We focus on preconditioned iterations applied on the equivalent real formulation (see, e.g., [6] and [7] for other formulations)

$$\begin{pmatrix} A & -B \\ B & A \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} b \\ c \end{pmatrix}$$

or

$$\mathcal{M}u = d$$

We exploit the similarities between the above form and generalized saddle point problems [3] to introduce and analyze several block preconditioners. In particular, we consider block diagonal, perturbations of block diagonal, block triangular as well as one based on symmetric and skew-symmetric splitting [2,5]

$$\begin{pmatrix} A & -B \\ B & A \end{pmatrix} = \begin{pmatrix} A & 0 \\ 0 & A \end{pmatrix} + \begin{pmatrix} 0 & -B \\ B & 0 \end{pmatrix},$$

or

$$\mathcal{M} = \mathcal{H} + \mathcal{S}.$$

Real formulations have been considered in several works (see, e.g., [1,4,6]) because there are several good packages which work in real arithmetics. It has been observed in [7] that, in general, iterative solvers show often an unfavorable convergence behavior for the transformed problem with respect to the original one. However, we experienced that preconditioning can be very beneficial.

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