On a Class of Toeplitz Matrices Related to Radial Functions

Dario Bini
bini@dm.unipi.it
Università di Pisa, Italy
A. De Rossi and B. Gabutti
Università di Torino, Italy

A function $r(x) : \mathbb{R}^n \to \mathbb{R}$ is a radial function if there exists $f(x) : \mathbb{R} \to \mathbb{R}$ such that $r(x) = f(||x||)$, where $||x||$ is the Euclidean norm of $x \in \mathbb{R}^n$. Given a radial function $r(x)$ and a set of knots $\{x_j \in \mathbb{R}^n,\ j = 1 : n\}$, we define the $n \times n$ matrix $A = (a_{i,j})$ associated with a differential operator L, say $L = \Delta = \frac{\partial^2}{\partial x_1^2} + \cdots + \frac{\partial^2}{\partial x_n^2}$, as $a_{i,j} = \Delta r(x - x_j)|_{x=x_i}$.

If the knots are displaced on a grid with constant stepsize then A is block Toeplitz with Toeplitz blocks.

We analize some spectral properties of the matrix A for the multiquadric function $r(x)$ such that $f(t) = (t^2 + c^2)^{1/2}$, where c is the shape parameter, and for the model problem where $L = \Delta$ is the Laplacian. For $n = 1$ we provide an expression for the symbol associated with A and evaluate the conditon number of A as a function of the shape parameter c. We discuss the problems encountered for $n > 1$.