Almost Block Diagonal Linear Systems

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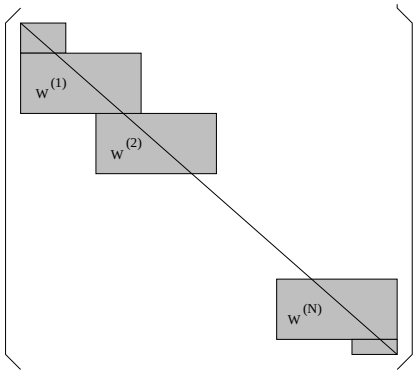
Almost block diagonal (ABD) linear systems arise in various numerical techniques for solving two–point boundary value problems for ordinary differential equations (BVODEs) with separated boundary conditions, and in related partial differential equations. The most frequently occurring ABD structure is shown in the figure, where the intermediate blocks $W^{(i)}$ are of equal size and the overlap between successive blocks is equal to the sum of the number of rows in the first and last blocks. This structure is exploited in algorithms that minimize fill-in and computational cost without compromising stability. Naive approaches to solving ABD systems involve considering them as banded or block tridiagonal systems. These approaches are undesirable for several reasons not the least of which is that they introduce fill-in when the structure is imposed on the system as well as in the solution procedure, leading to significant inefficiencies.



We outline how ABD systems arise when solving BVODEs using three basic techniques – finite differences, orthogonal spline collocation and multiple shooting – and describe efficient algorithms for their solution. Also, we discuss systems of similar structure arising in the solution of elliptic and parabolic partial differential equations, and overview methods for solving bordered ABD systems, which arise in solving BVODES with non-separated boundary conditions.