Superlinear Convergence for PCG Method using Algebra plus Band Preconditioners

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This paper concerns on the fast and efficient solution of $n \times n$ symmetric ill conditioned Toeplitz systems $T_n(f)x = b$, where the generating function f is a priori known, nonnegative real valued, having isolated roots of even order. The preconditioner setting that we propose is a product of a band Toeplitz matrix and matrices that belong to any trigonometric algebra. The proposed scheme tries to embody the well known advantages that each of the components of the present product when it is used alone. As a result we obtain a flexible preconditioner which can be applied to the system $T_n(f)x = b$ infusing superlinear convergence to the PCG method. The important feature of this proposal is that it can be easily extended to cover the *2D* case i.e. ill conditioned band Toeplitz with Toeplitz blocks (BTTB) matrices. Finally, we compare our method with the already known in the literature techniques. The results fully confirm the effectiveness of the proposed strategy and the adherence with the theoretical analysis.