Circurlant-Plus-Low-Rank Approximations and Circulant Preconditioners

Ivan Oseledets
Russian Academy of Science, Russian

In this talk the unifying approach for the construction of circulant preconditioners for a given (Toeplitz) matrix will be described. It is based on the observation that the existence of the eigenvalue cluster of a preconditioned matrix AP^{-1} at unity is directly related to the decomposition of form

$$A = P + R + E,$$

where P is a preconditioner, R is "small rank matrix" and E is a "small norm matrix".

Restricting P to some matrix class (for example, circulant) we obtain a matrix approximation problem: approximate a given matrix by a sum of structured and low-rank matrix.

It will be shown that the solution of this approximation problem can be performed fast and efficiently using "black dot algorithm" (I. Oseledets, E. Tyrtyshnikov). In contrast to several "function-based" circulant preconditioners used for "bad" symbols, it is constructed entirely from the entries of a given matrix and performs equally as the best of the known or better than those for the same symbols.

Also we will briefly discuss the generalization of the "black dot algorithm", its relation to the minimal rank completion problem and how can it be used to estimate missing entries in a large arrays of data.

This is a joint work with Eugene Tyrtyshnikov.