Preconditioners for Dense and Ill-conditioned Saddle Point Systems Arising from Interior-point Methods for Convex Quadratic Semidefinite Programming

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We consider a primal-dual path-following Mehrotra-type predictor-corrector method for solving convex quadratic semidefinite programming (QSDP) problems of the form: $\min_X\{\frac{1}{2}X \bullet Q(X) + C \bullet X : A(X) = b, X \succeq 0\}$, where Q is a self-adjoint positive semidefinite linear operator on S^n, $b \in R^m$ and A is a linear map from S^n to R^m. At each interior-point iteration, the search direction is computed from a dense symmetric indefinite linear system (called the augmented equation) with dimension $m + n(n+1)/2$. Such linear systems are very large when n is larger than a few hundreds and can only be solved by iterative methods. We propose three preconditioners for the augmented equation, and show that the corresponding preconditioned matrices have favorable asymptotic eigenvalue distributions for fast convergence under suitable nondegeneracy assumptions. We are able to solve the augmented equation efficiently via the preconditioned symmetric quasi-minimal residual iterative method with the preconditioners constructed. Numerical experiments on a variety of QSDPs with matrices of dimensions up to $n = 1600$ are performed and the computational results show that our methods are efficient and robust.