Rank Structured Matrix Operations

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We call a matrix *rank structured* if the ranks of certain submatrices starting from the lower-left matrix corner, as well as the ranks of certain submatrices starting from the upper-right matrix corner, are small compared to the matrix size. The class of rank structured matrices contains as special cases the classes of semiseparable matrices, unitary Hessenberg matrices, quasiseparable matrices and so on.

In order to specify algorithms for the class of rank structured matrices, we will first need an efficient representation. To this end we will use the Givens-weight representation: this is a generalization of the Givens-vector representation for semiseparable matrices [5], which is generalized to the case of an *arbitrary* rank structure.

Using the Givens-weight representation, we can devise an efficient algorithm to transform a rank structured matrix into a Hessenberg matrix by the use of unitary similarity transformations. The algorithm is important since the Hessenberg reduction process is commonly used as a first step to determine the eigenvalue spectrum of a matrix. We also show how the algorithm can be modified to transform the given matrix to bidiagonal form, by means of possibly different unitary row and column operations, i.e. by a reduction of the form $A \mapsto UAV$. This reduction can be used as a first step to compute the singular value decomposition of a matrix.

When investigating matrix structures preserved under the QR-algorithm, under matrix inversion, under Schur complementation, the class of rank structured matrices turns out to maintain its structure under these operations [1-4]. Taking the structure of the Q and R factor of the QR-factorization of a rank structured matrix into account, an efficient algorithm can be designed to compute this QR-factorization and solve the corresponding system of linear equations. These properties can also be used to devise a QR-algorithm where each step of the algorithm needs $O(n)$ floating point operations on a rank structured matrix of size n. Also a rank structured representation for the inverse can be computed in an efficient way.

As far as time permits one or more matrix operations on rank structured matrices will be explained in detail.

References

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