Alignment Matrix and the LTSA Algorithm for Nonlinear Dimensionality Reduction

Qiang Ye
qye@ms.uky.edu
Department of Mathematics, University of Kentucky, USA

Given a set of high-dimensional data points, the goal of dimensionality reduction is to find a low-dimensional parametrization for them. Usually it is easy to carry out this parametrization process within a small region to produce a collection of local coordinate systems. Alignment is the process to stitch those local systems together to produce a global coordinate system.

The so-called alignment matrix is one assembled from smaller matrices that are projectors on subspaces. It lies at the center of the Local Tangent Space Alignment method (LTSA) for nonlinear dimensionality reduction recently proposed by Zhang and Zha. In this talk, we present some some theoretical results characterizing the null space and the smallest non-zero eigenvalue of the alignment matrix. Our analysis provides a theoretical justification for the LTSA algorithm and shows that LTSA recovers locally isometric embedding up to a rigid motion.

This talk is based on joint works with H. Zha and R. Li.